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THE PARETIAN OPTIMUM CONDITIONS OF WELFARE MAXIMIZATION AS A GUIDE TO POLICY

by

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ABSTRACT

The attainment of a Paretian welfare optimum is possible if, and only if, a certain set of marginal stability and total conditions are simultaneously satisfied. In the past, these marginal conditions and, in particular, their practical interpretations, such as the price=marginal cost condition, have been used by welfare economists as criteria for the attainment of, or a movement towards, a position of maximum community welfare.

This thesis sets out these optimum conditions and examines their practical applicability in the real world. The operation of numerous constraints is seen, in practice, to preclude the attainment of a Paretian optimum. The problem of maximizing social welfare is thus found to be similar to several other maximization problems in a constrained setting, and to involve the determination of a second best position.

The first-order conditions defining the second best are, however, either too complex to be of any value or are indeterminate. The second-best optimum is thus also useless. It is concluded that the originally defined marginal conditions might still be of limited value in some cases in guiding policy towards the obtainable optimum.

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CHAPTER I

THE CONDITIONS FOR A PARETIAN OPTIMUM

The concept of a Paretian optimum has long been used by welfare economists to define the position of maximum welfare or satisfaction for any particular community[1] of producers and consumers. The attainment of a Paretian optimal position is possible if, and only if, a certain set of first-order "marginal conditions" are simultaneously fulfilled along with the satisfaction of a set of second order "stability conditions", and is ensured subject to the satisfaction of a set of "total conditions." The purpose of this thesis is to set out the development of, and the refinements made to, the conditions of a Paretian optimum; to discuss the applicability of these Paretian optimum conditions with respect to practical policy prescription; in particular, to trace the developments which culminated in the "General Theory of Second Best"; and, finally, to review the more pragmatic

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[1]The question whether or not the concept of a Paretian optimum can be applied in a socialist economy has been discussed by many writers among whom are A. P. Lerner in "Economic Theory and Socialist Economy," Review of Economic Studies, Vol. II (1934-35), pp. 51-61, and by Maurice Dobb in "Economic Theory and Socialist Economy: A Reply," Review of Economic Studies, Vol. II (1934-35), pp. 145-51.

approaches to practical policy under the heading of "third-best."

The phrase "Paretian optimum" takes its name from Vilfredo Pareto. Pareto, conceiving economic equilibrium as best described by the fundamental opposition of tastes or ophelimities to obstacles,[2] had attempted to liberate the indifference curve analysis from its hedonistic and utilitarian aspects. In contrast to Edgeworth, who had first introduced the concept of indifference curves as derivable from a given utility function,[3] Pareto began first with the notion of indifference curves, a set of which could be found through empirical research; and from this set of indifference curves, he developed a function representing an indexed surface of utility.[4] The various indifference curves making up such a function could then be given indices which did not necessarily measure different degrees of utility represented by the curves.[5] The

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[2]These obstacles of the "first" and "second" orders are set out in detail by Umberto Ricci in "Pareto and Pure Economics," Giornale degli Economisti, 1924; reprinted in Review of Economic Studies, Vol. I (1933-34), pp. 16-17.

[3]F. Y. Edgeworth, Mathematical Psychics (London: Kegan Paul Co., (1881), p. 21.

[4]Vilfredo Pareto, "Mathematical Economics," International Economic Papers, No. 5, 1955, pp. 58-102. There is an excellent discussion of Pareto's indifference curve analysis by Henry Schultz in The Theory and Measurement of Demand (Chicago: University of Chicago Press, 1938), pp. 5-58.

[5]That the assumptions made by Pareto in his indifference curve analysis do in fact imply measurability, is maintained

importance of the indifference curve analysis to welfare economics is such as to have led K. E. Boulding to state that "it is only a slight exaggeration to claim that modern welfare economics has developed largely as a result of the invention of this powerful analytical tool."[6] Now although the indifference curve will be used later on in this chapter to illustrate the marginal conditions of a Paretian optimum, it is not with this contribution of Pareto to welfare economics that this thesis is primarily concerned. But rather, it is concerned with the particular definition of the position of maximum satisfaction which Pareto gave to the study of welfare.

Introducing the term "ophelimity"[7] as an alternative to tastes or satisfactions, Pareto defined maximum ophelimity or maximum satisfaction as follows:

> Let us begin by defining a term which is very convenient to use in order to save words. We shall say that the individuals of a group in a given position have maximum ophelimity if it is impossible to depart some small distance from this position in such a way that this departure is useful for all individuals of the group. Every small displacement from this position would necessarily have the effect of being useful to some of the individuals of the society and detrimental to some others.[8]

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by O. Lange in "The Determinateness of the Utility Function," Review of Economic Studies, Vol. I (1934), pp. 218-25; and by H. Bernardelli in "A Note on the Determinateness of the Utility Function," Review of Economic Studies, Vol. II (1934-35, pp. 69-75.

[6]K. E. Boulding, "Welfare Economics," in A Survey of Contemporary Economics, Vol. II, ed. B. F. Haley, (New York: Blakiston, 1952), p. 2.

[7]Ricci, loc. cit., pp. 13-14, and p. 19.

[8]Vilfredo Pareto, Manuel d'Economie politique, (1909),

Because of Pareto's habit of working fast and not concerning himself with details, the phrase, "useful for all individuals of the group" may be interpreted, according to Professor Gustavo Del Vecchio and Ragnar Frisch, as "useful to at least one of them and for all the rest of them either useful or indifferent."[9]

Maximum social ophelimity, according to Pareto, could then be attained under the exchange conditions of perfect competition in which, following Walras, price equaled marginal cost and in which costs in a technical sense were at a minimum; this last condition was derived by Pareto as a result of his mathematical analysis.[10] Although it is not certain just how strongly Pareto, himself, believed that the régime[11] of free competition led to maximum satisfaction, it does not appear that Pareto had anywhere proved in a

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p. 337, as translated and quoted by Ragnar Frisch in "On Welfare Theory and Pareto Regions," International Economic Papers, No. 9, 1959, p. 91. It should be noted that a small displacement could be detrimental to all the individuals of the society.

[9]Frisch, loc. cit., pp. 91-2.

[10]Ricci, loc. cit., p. 18. Whether the costs were average or marginal was not made explicit by the early writers on the subject of the conditions defining a social maximum.

[11]According to Frisch, loc. cit., pp. 40-41, a régime is the complete set of conditions, including the objective conditions comprised of obligatory and facultative ones and the conjectural conditions, which the variables of any decision model will satisfy under any given economic policy or policies.

mathematically rigorous manner that free competition produces maximum satisfaction.[12]

Concerning himself with the manner in which the Ministry in charge of production in a socialist state could secure the maximum advantage from the resources at its disposal, E. Barone attempted to set out the conditions of maximum collective welfare using only "the old and simple ideas of demand, supply and cost of production" and without having recourse to either "the concepts of utility, of the final degree of utility and the like," or to "Pareto's concept of the Indifference Curve."[13] He first defined the maximum of free competition in much the same way as Pareto had defined the position of maximum welfare, but Barone supplemented the Pareto-type definition with an exceedingly early but yet precise formulation of a compensation principle, as follows:

> The maximum, we repeat, simply means this: that by substituting other conditions for one or more of the characteristics of free competition (minimum costs of production, equality of prices and costs of production) the conditions of all could not be improved. On the contrary, if some are benefited by this substitution, their gain is less than the loss of those who suffered. So that if all their gain is taken from those who gained by substitution, and is given to those who suffered loss by it, the latter could never retrieve their former position and some would always remain losers.[14]

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[12] Frisch, loc. cit., p. 92.

[13] E. Barone, "The Ministry of Production in a Socialist State," Giornale degli Economisti (1908); collected and translated in Collectivist Economic Planning, ed. F. A. von Hayek, (London: Routledge and Sons, Ltd., 1935), p. 246.

[14] Ibid., p. 257.

The maximum of free competition was thus thought to be attained when the costs of production were minimized in a technical sense and prices made equal to marginal costs of production. These two conditions, then, are the same as those set out by Pareto for the attainment of a Paretian optimum. Barone then succeeds in proving to his own satisfaction that the same two conditions which characterize the maximum of free competition also characterize, and are necessary for, the attainment of the maximum collective welfare.[15]

In any discussion of the early development of the conditions necessary for the attainment of a position of maximum welfare or satisfaction, mention should be made of the work of A. Marshall and his belief in the "doctrine of maximum satisfaction." This phrase is defined as "the doctrine that the free pursuit by each individual of his own immediate interest, will lead producers to turn their capital and labour, and consumers to turn their expenditure into such courses as are most conducive to the general interests."[16] Subject to the assumption that "equal sums of money measure equal utilities to all concerned,"[17] and the assumption that "every fall in the price which producers receive for the commodity, involves a corresponding loss to them,"[18] that is, ignoring the effects

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[15] Ibid., p. 274.

[16] A. Marshall, Principles of Economics (8th ed.; London: Macmillan and Co. Ltd., 1930), p. 502.

[17] Ibid., p. 470.

[18] Ibid., p. 471.

of improvements in organization, Marshall held that a position of maximum satisfaction resulted when equilibrium between demand and supply was attained; and, in particular, maximum satisfaction resulted when the marginal utility of what each consumer or producer received was equal to the marginal utility of what he gave up in exchange.[19] Furthermore, Marshall felt there was a "constant tendency to make H-V a maximum for the society as a whole,"[20] where "H" is the sum total of satisfactions and "V" is the sum total of dissatisfactions, both of which, satisfactions and dissatisfactions, accrue to the community as a result of economic factors. In sum, Marshall believed that there was a constant tendency for the freely competitive society to drift towards the position of maximum net satisfaction. Accordingly, the conditions of free competition, defined in terms of marginal utilities and disutilities, were also the conditions necessary for the attainment of maximum welfare.

The contribution[21] of A. C. Pigou to the development of the conditions describing a position of maximum welfare is an elaboration of the work of Marshall and appears to be at

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[19] Ibid., p. 470.

[20] Ibid., p. 851.

[21] A. C. Pigou, The Economics of Welfare (4th ed.; London: Macmillan and Co. Ltd., 1932). Some writers suggest that Pigou's work is not part of the mainstream of the development of modern welfare economics, indeed, that it leads to a "blind alley." See Boulding, loc. cit., pp. 1-2.

least two-fold. In the first place, Pigou clearly distinguishes between marginal social net product and marginal private net product. The marginal private net product is defined as "that part of the total net product of physical things or objective services due to the marginal increment of resources in any given use or place which accrues in the first instance--i.e. prior to sale--to the person responsible for investing resources there";[22] while the marginal social net product is defined as "the total net product of physical things or objective services due to the marginal increment of resources in any given use or place, no matter to whom any part of this product may accrue."[23] Accordingly, using this new terminology the national dividend is maximized, and, thereby, economic welfare is also maximized,[24] when the arrangement of resources is such that the values of the marginal social net products are equal in all uses.[25] In particular, ideal output, that which maximizes the national dividend, in a many-firm industry is attained at that level of output "which makes the demand price of the output equal to the money value of the resources engaged in producing a marginal unit of output; in other words, it will be the output that makes the demand price and the marginal supply price to the community equal."[26]

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[22] Ibid., pp. 134-5.

[23] Ibid., p. 134.

[24] Ibid., p. 31.

[25] Ibid., p. 136.

[26] Ibid., pp. 803-4.

The second contribution of Pigou would be his explicit recognition of the fact that the satisfaction of the several marginal equalities would not by itself ensure the attainment of a position of maximum satisfaction, that is, ensure that the society had reached the top of the highest mountain rather than the top of any particular mountain. An extensive quotation clearly illustrates Pigou's discussion of the attainment of an absolute maximum in contrast to a relative maximum, as follows:

> But when the values of the marginal social net products in all uses are equal, the dividend <u>need not</u> attain an unequivocal maximum. For, if several arrangements are possible, all of which make the values of the marginal social net products equal, each of these arrangements does, indeed, imply what might be called a <u>relative maximum</u> for the dividend; but only one of these maxima is the unequivocal, or absolute, maximum. All of the relative maxima are, as it were, the tops of hills higher than the surrounding country, but only one of them is the highest hill-top of all. Furthermore, it is not necessary that all positions of relative maximum should represent larger dividends than all positions which are not maxima. On the contrary, a scheme of distribution approximating to that which yields the absolute maximum, but not itself fulfilling the condition of equal marginal yields, would probably imply a larger dividend than most of the schemes which do fulfill this condition and so constitute relative maxima of a minor character. A point <u>near</u> the summit of the highest hill may be higher than any summit except the highest itself.[27]

A further condition describing the position of maximum welfare can thus be implied from Pigou's analysis. In a negative form it would state that it must be impossible to secure benefit by "a temporary bounty (or temporary protection) so arranged as to jerk the industrial system out of its present poise at

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[27] <u>Ibid.</u>, p. 140.

a position of relative maximum, and induce it to settle down again at the position of absolute maximum--the highest hill-top of all."[28]

Writing in 1934[29] about monopoly power, its measurement and the social loss to society resulting from the existence of monopoly, A. P. Lerner added definite refinement to the conditions which must necessarily be fulfilled in order that a relative optimum be attained. Using the concept of indifference curves along with the notion of a "displacement cost curve," defined as the "productive indifference curve"[30] of the whole community, Lerner defines the optimum resulting from the satisfaction of certain inclusive conditions as follows:

> If both of these conditions are satisfied, as between each individual's indifference curves and the communal displacement costs curve on the one hand, and as between each individual's indifference curves and every other individual's (inverted) indifference curves, on the other hand, it is impossible to improve the position of any individual without worsening the position of some other individual. The optimum position, relative to the distribution of income between individuals, is attained.[31]

These conditions call for a tangency solution[32] between the

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[28] Ibid., p. 141.

[29] A. P. Lerner, "The Concept of Monopoly and the Measurement of Monopoly Power," Review of Economic Studies, Vol. I (June, 1934), pp. 157-75.

[30] Ibid., p. 163.

[31] Ibid., p. 165.

[32] As Lerner himself states in the text of his article, ibid., p. 164, an actual tangency solution is not strictly required.

indifference and transformation curves or between two sets of indifference curves belonging to two different individuals. Stated negatively, they imply that it is not possible to increase production, thereby raising the potential levels of consumption for one or more persons nor is it possible, maintaining production levels constant, to so reorganize the allocation of goods among individuals in such a way as to move one or more individuals to a preferred position without moving any other individual or individuals to a worse position.

Further detail and precision is given to the conditions describing a position of maximum welfare when Lerner in his book, The Economics of Control,[33] defines the optimum allocation of factors of production, among different uses, by his five "welfare equations." The first equation requires the equality of the marginal social benefit, defined as "the benefit to society (i.e. the net benefit to all members of the society affected) from the particular increment of output of the product considered," with the value of the marginal product, defined as "the physical increment of output of the product being considered, multiplied by the price paid for it by the consumer." The equality of the value of the marginal product with the marginal private revenue, defined as "the increase in revenue (positive or negative) received by the producer as a result of producing and selling the increment in output," is required by the second equation. The third

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[33] A. P. Lerner, The Economics of Control (New York: The Macmillan Co., 1944).

welfare equation calls for profit maximization and, thereby, the equality of marginal private revenue with marginal private cost, defined as "the increase in cost incurred by the producer as a result of increasing the quantity of the factor he purchases in order to be able to produce the increment of output." The fourth equation, then, necessitates the equality of the marginal private cost with the value of the marginal factor, here defined as "the physical increment of the factor of production (that is needed to make the increment of product) multiplied by the price per unit paid for it and received by the owner of the factor." And finally, the fifth equation sets out the necessary equality of the value of the marginal factor with the marginal social cost, defined as "the sacrifice to society from having the marginal factor used up here so that it is not available for use elsewhere." These five welfare equations are concisely summarized as follows:

msb = vmp = mpr = mpc = vmf = msc.[34]

The important condition for the optimum allocation of factors is, thus, given as the necessity of equating the marginal social benefit from an increase in output with the marginal social cost of using up factors of production in order to increase output. In his concern for the marginal social benefit as opposed to the marginal private revenue, Lerner follows closely the example of Pigou.

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[34]These above-mentioned equations and definitions are taken from Lerner, loc. cit., pp. 75-7.

Instead of concentrating on the adjustment resulting from a change of one unit of a certain factor of production, it is equally satisfactory to concentrate on the adjustment resulting from a change of one unit of a certain product produced; that is, a position of maximum welfare could be set out by defining the optimum levels of production of the various products. In this case, the welfare equations could be summarized as follows:

msb = p = mpr = mpc = vmf = msc,[35]

where p stands for the price of the newly produced unit and where the marginal social benefit and cost concepts refer to the benefits and costs accruing to society as a result of increasing the production of a particular product by one unit.

The form in which the conditions describing a position of maximum welfare are usually presented varies slightly from the form in which they were presented above by Lerner. Rather, these conditions are set out in a manner which elaborates on the presentation of them by J. R. Hicks in his article, "The Foundations of Welfare Economics."[36] According to Hicks, a position of welfare optimum, with respect to the organization of resources, is attained subject to a set of general conditions made up of the "marginal

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[35] Ibid., pp. 96-7.

[36] J. R. Hicks, "The Foundations of Welfare Economics," Economic Journal, Vol. 49 (December, 1939), pp. 696-712.

conditions," the "stability conditions," and the "total conditions."

The marginal or first order conditions require that "the marginal rate of substitution between any two commodities must be the same for every individual (who consumes them both) and for every producing unit (which produces them both) in the whole economy."[37] In an analogous manner, these conditions must hold as between any two factors of production and as between any factor of production and the product it produces.

The stability or second order conditions, dealing with the curvature of the various indifference and transformation curves, exclude the possibility of a minimum and ensure the attainment of a maximum position. Like the marginal conditions, the stability conditions are necessary conditions. In the Pareto-Edgeworth box diagram of Figure 1, the stability conditions have the force of assuming the continuity of all preference and transformation functions in the relevant area under discussion. Specifically, these conditions require that the transformation curves must be concave to the origin, thereby, implying a diminishing marginal rate of transformation;[38] and, that the indifference curves be

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[37] Ibid., p. 704.

[38] J. R. Hicks, Value and Capital (2nd ed.; Oxford: Clarendon Press, 1946), p. 21.

convex to the origin, or at least less concave to the origin than the transformation curves,[39] thereby, implying a diminishing marginal rate of substitution.[40]

Taken together, the marginal and stability conditions would guarantee the attainment of a relative optimum. However, in order that an absolute maximum be attained, it is necessary to invoke the total conditions. The total conditions, then, aim at solving the problem of relative maxima posed earlier by Pigou's work; that is, they ensure "that no improvement can be brought about by the complete abandonment of the production or consumption of some one commodity, either in one producing or consuming unit, or generally; and that no improvement can be secured by the introduction of new commodities, which could have been produced or consumed, but were not being produced or consumed, either partially or generally, in the initial situation."[41] To complete this set of total conditions it is necessary to add the condition that a reallocation of the quantities produced of the various commodities would not lead to the attainment of an increased welfare position. In particular, the total conditions must hold for all the various factors of production, especially labour. It might be noted here, that these total conditions would appear to lack operational meaning. According to

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[39] M. W. Reder, Studies in the Theory of Welfare Economics, (New York: Columbia University Press, 1947), p. 37, n. 12.

[40] Hicks, Value and Capital, p. 81.

[41] Hicks, "The Foundations of Welfare Economics," p. 704.

P. A. Samuelson, an operationally meaningless theory is one "which has no empirical implications by which it could conceivably be refuted under ideal empirical conditions."[42] To this definition could be added the condition that a meaningful theory must be capable of being proved correct. For it is in this sense that the total conditions are meaningless; they are merely of descriptive value in defining a position of maximum welfare as there is no possible way of telling whether or not the introduction of a new product is, or was, possible and if so whether or not welfare would, or had, increased as a result. It is, however, with the marginal conditions that this paper is primarily concerned. Accordingly, it will be advantageous to set out the marginal conditions in more detail than has been done above and in the form in which they are usually presented by modern-day economists.[43]

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[42] P. A. Samuelson, "The Empirical Implications of Utility Analysis," Econometrica, Vol. 6 (October, 1938), p. 344.

[43] For examples of such a presentation see A. Bergson, "A Reformulation of Certain Aspects of Welfare Economics," Quarterly Journal of Economics, Vol. 52 (February, 1938), pp. 310-34; Boulding, loc. cit., pp. 14-23; J. E. Meade, Trade and Welfare (London: Oxford University Press, 1955), pp. 51-67; I. M. D. Little, A Critique of Welfare Economics (2nd ed.; Oxford: Clarendon Press, 1957), pp. 128-51; especially Reder, loc. cit., pp. 21-36; and, for mathematical exposition, O. Lange, "The Foundations of Welfare Economics," Econometrica, Vol. 10 (July, 1942), pp. 215-28; and P. A. Samuelson, The Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947), pp. 229-49. External economies and diseconomies are included in the mathematical presentation of the marginal conditions by Lange, loc. cit., by Samuelson, loc. cit., and by Gerhard Tintner, "A Note on Welfare Economics," Econometrica, Vol. 14 (January, 1946), pp. 69-78.

The first marginal condition, refering to the optimum allocation of products, may be set out as:

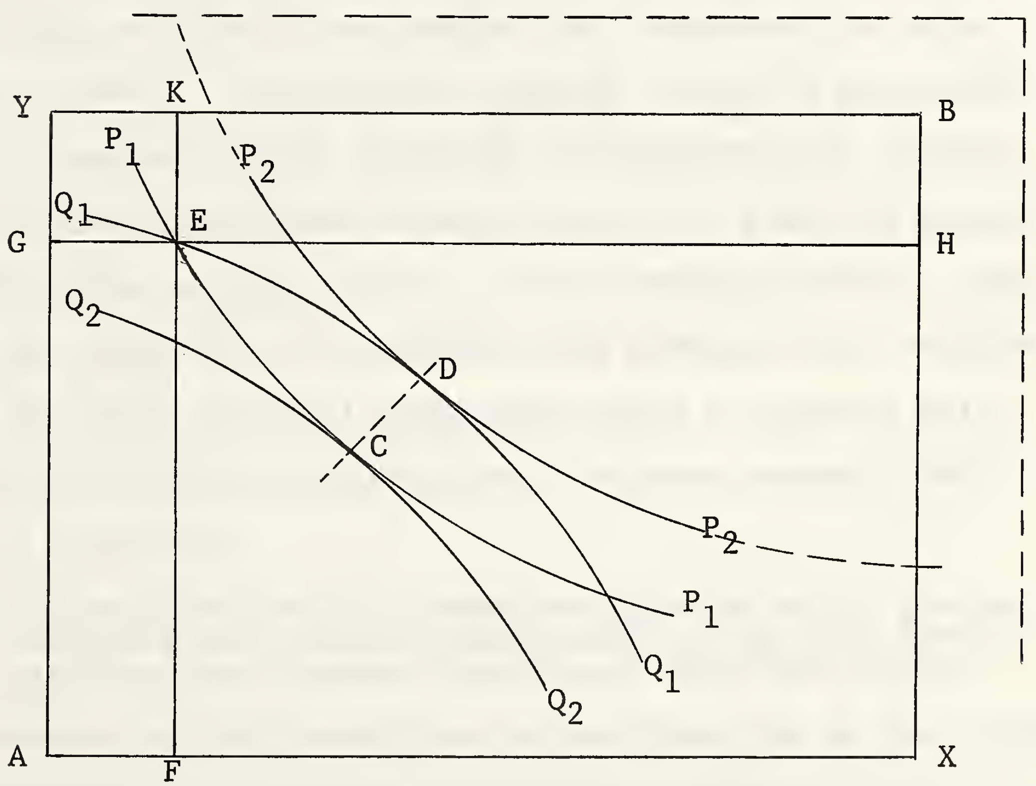
> the marginal rate of substitution between any two products must be the same for each and every individual who consumes both of these products.

This first marginal condition is illustrated diagramatically by the Pareto-Edgeworth box-diagram of Figure 1. In Figure 1, the rectangle AYBX is so drawn as to make AY represent the total amount of product Y available in the economy for consumption by the two individuals A and B and to make AX represent the total amount of the product X available for consumption. The indifference curves P_1P_1 and P_2P_2, drawn convex to origin A, represent part of the preference pattern of individual A for the two goods X and Y; while, in a similar manner, the indifference curves[44] Q_1Q_1 and Q_2Q_2, drawn convex to the inverted origin B, represents part of the preference of individual B for the two goods X and Y. Any point within the box diagram represents a particular distribution of the goods X and Y between the two individuals A and B. Accordingly, the point E represents that particular distribution of goods which allocates AF of good X and AG of good Y to A, and which allocates FX of good X and GY of good Y to B. Clearly, a movement from point E to any position inside of the area bounded by the indifference curves Q_1Q_1 and P_1P_1 will make both individuals

[44] In the analysis of Little, these are called behavior lines, loc. cit., p. 131, following his development of the behavior line analysis on the foundations of revealed preference theory, ibid., pp. 283-9.

better off in the sense of raising them to higher indifference levels. A maximum position is reached, however, at the tangency solution of the indifference curve of A with that of B, such as the point D or the point C. Any point on the "contract curve,"[45] defined as the locus of the several tangency solutions of the indifference curves of A with those of B, through C and D will yield a position of maximum welfare as far as the allocation of products between individuals is concerned.

Figure I.--Pareto-Edgeworth Box Diagram



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[45]Edgeworth, loc. cit., p. 25 et seq.

Refering to the optimum allocation of products among various producers, the second marginal condition may be stated as:

> the marginal rate of transformation in the production of any two products must be the same for each and every pair of firms which produce both of these products.

This marginal condition may also be illustrated diagramatically by Figure 1. If Q_1Q_1 is renamed as the transformation curve of "firm" A and P_1P_1 is renamed the transformation curve of "firm" B, and if the initial position is given as E, it is possible to increase the total output of both products X and Y by moving to some point such as D on the transformation curve Q_1Q_1 of firm A but inside the transformation curve P_1P_1 of firm B. Accordingly, maximum levels of production can be reached only by shifting the production of commodities to the lowest cost producer until the point is reached at which the marginal rates of transformation between products are equal for all producers who produce these products.

The third marginal condition refers to optimum allocation of factors of production as between products and it may be stated as:

> the marginal rate of transforming a factor of production in a particular product must be the same for all producers who produce the product with that factor.

The essence of this condition is the direction of the factors of production to the high productivity producer, that is, to the production unit wherein the marginal physical product of the particular factor in the production of the particular product is highest.

The fourth marginal condition refers to the substitution of the various factors of production in the producing of a particular commodity and may be set out as follows:

> the marginal technical rate of substitution between any two factors both of which are used in the production of a particular product must be the same for every production unit which employs the two factors to produce that particular product.

In Boulding's terminology, the "marginal rates of equal-product substitution of two factors"[46] must be the same for every pair of producers.

The fifth marginal condition combines technical efficiency in production with optimum allocation of products among consumers according to their individual preference patterns and may be stated as:

> the marginal rate of substitution between any two products for every individual consuming both must be equal to the marginal rate of transformation between these two products for all producers who produce both products.

This condition ensures that production conforms to consumers' preference patterns.

The sixth marginal condition refers to the degree of intensity with which any particular factor of production is used and, in particular, refers to the relationship between leisure and product. This sixth condition may be stated as:

> the marginal rate of substitution between the reward of the factor and the time spent in productive use must be the same as the marginal rate of transformation between the product produced and the time spent

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[46]Boulding, loc. cit., p. 21.

> by the factor unit in its production for any given factor owner and producing unit.

This condition simply states that each factor of production must be paid the value of its marginal physical product and that the price of the factor must be equal to the marginal supply price of the factor.

Inter-temporal considerations,[47] given the absence of uncertainty, are dealt with by the seventh marginal condition. It may be stated as:

> the marginal rate of substitution between any two resources, either factors or products, must be the same for all individuals or firms who hold control over both resources over time.

It is expedient to assume that over time a particular factor or product changes; thus, a pound of coal today is tomorrow considered to be a different factor input. This seventh condition is primarily concerned with lending and borrowing and requires that the rate of substitution over time of any two individuals for a particular product or factor must be the same.

These seven marginal conditions[48] have been summarized

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[47]The problems, associated with the optimum allocation of resources over time, of changing consumer preferences and of imperfectly foreseen future prices and production possibilities are dealt with by E. Malinvaud, "The Analogy Between Atemporal and Intertemporal Theories of Resource Allocation," Review of Economic Studies, Vol. 28 (1961), pp. 143-60. The importance of the possible link between current patterns of resource use and future patterns of demand is discussed by W. Peters, "A Note on Welfare Economics and Demand Links over Time," The Manchester School, Vol. XXX (May, 1962), pp. 181-86.

[48]Boulding, ibid., p. 23, includes two conditions relating to uncertainty and liquidity; namely, that the

by Reder as follows:

1. The marginal rate of substitution between any two products must be the same for every individual who consumes both.

2. The marginal rate of transformation between any two products must be the same for any two firms that produce both.

3. The marginal rate of transformation between any factor and any product must be the same for any pair of firms using the factor and producing the product.

4. The marginal technical rate of substitution between any pair of factors must be the same for any two firms using both to produce the same product.

5. The marginal rate of substitution between any pair of products for any person consuming both must be the same as the marginal rate of transformation (for the community) between them.

6. The marginal rate of substitution between the amount of (product) X received for aiding in its production (by a given firm) and the time spent in rendering this aid must be the same for each factor unit owner as the marginal rate of transformation between the time his factor unit spent in aiding production (in this way) and (the product) X.

7. The marginal rate of substitution between resource control at any pair of moments (ti and tj) must be the same for every pair of individuals or firms (including pairs, one member of which is a firm and the other an individual).[49]

In Boulding's work, these seven conditions are reduced to two grand conditions as follows:

(i) wherever transformation of one variable into another is technically possible, the rate of

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marginal rates of substitution of all individuals for assets of different degrees of uncertainty and liquidity must be equal.

[49]Reder, loc. cit., pp. 35-6.

indifferent substitution (that is, the amount of one variable which can be substituted for one unit of the other without feeling a gain or loss) must be equal to the rate of technical substitution--that is, the amount of the first variable which can be obtained technically by giving up one unit of the second;

(ii) all equivalent rates of technical and indifferent substitution must be equal.[50]

Carrying the process of summarizing the necessary marginal conditions a step further is the one all-inclusive condition set out by Samuelson as follows:

> Necessary Marginal Conditions for an Optimum. Between any two variables, the marginal rates of substitution must be (subjectively) equal for all individuals, and (technically) equal for all alternative processes, with the common technical and subjective ratios being equivalent; otherwise there exists a physically attainable position that makes everyone better off.[51]

As in the two grand conditions of Boulding, so also in Samuelson's "grand" condition variables are defined as inputs and outputs; and, in order to include time considerations, the variables may be taken to represent the same physical good which is considered to constitute different commodities in the various time intervals.[52]

In the above brief review of the marginal conditions, necessary for attainment of a Paretian optimum, it has been assumed that each individual has a utility function

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[50] Ibid., p. 23.

[51] P. A. Samuelson, "Welfare Economics: A Comment" in A Survey of Contemporary Economics, Vol. II, ed. B. F. Haley, (New York: Blakiston, 1952), p. 38.

[52] Lange, loc. cit., pp. 226-7.

which orders his preferences and that the state of technology remains unchanged, and, thereby, that each firm's productive capacities are determined by a given transformation function. Given these assumptions, then the marginal conditions are directly derivable from the definition of the Paretian optimum, that being the position from which it is impossible to move in a positive direction where a positive direction is defined to be a reorganization in which no individual is made worse off while at least one individual is made better off.[53] It should be noted, however, that the Paretian optimum as defined by the three sets of conditions, the marginal, stability and total conditions, is the position of maximum welfare subject to the given distribution of income. These three sets of conditions do not define a single point but rather define a limiting range of points, making up the "contract curve," each of which differs from the others according to the particular income distribution.[54]

In order to specify the particular Paretian optimum which is the optimum optimorum corresponding the preference patterns and, in particular, to the equity considerations, held by any particular society, several attempts have been made to construct a social welfare function. In this line

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[53]Otto Eckstein, "A Survey of the Theory of Public Expenditure Criteria," in Public Finances: Needs, Sources, and Utilization (National Bureau of Economic Research, Princeton: Princeton University Press, 1961), p. 441.

[54]Boulding, loc. cit., pp. 13-4.

of endeavor, the work of Bergson[55] and Samuelson[56] is exemplory. Recently, K. J. Arrow has set out his "General Possibility Theorem"[57] which most effectively points out the difficulties involved in developing an acceptable social welfare function.[58] In this thesis, the primary concern is that of establishing a Paretian optimum and of examining the conditions of such an optimum for the purpose of developing criteria for policy prescription. Accordingly, the question of specifying a particular point on the contract curve, that is, of specifying a particular distribution of income will not be dealt with.

In this first chapter, then, a position of maximum welfare has been defined as that situation in which "the productive resources are utilized in such a way that it is impossible to make any one person more satisfied (put on a higher indifference surface) without making at least one other person less satisfied (put on a lower indifference surface)."[59] The several conditions defining this position of maximum welfare have also been set out.

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[55]Bergson, loc. cit., 310-34.

[56]Samuelson, Foundations of Economic Analysis, pp. 203-53.

[57]K. J. Arrow, Social Choice and Individual Values, (New York: John Wiley and Sons, Inc., 1951).

[58]An exhaustive survey and evaluation of the several attempts to develop an acceptable social welfare function, where acceptibility is judged according to the criteria of consistency, power, and relevance, is given by J. Rothenberg in The Measurement of Social Welfare (Englewood Cliffs, N.J.: Prentice-Hall Inc., 1961).

[59]Reder, loc. cit., p. 15.

The purpose of the next chapter will three-fold: to discuss the various causes of divergencies between the marginal conditions; to examine the compatibility of the marginal conditions for a Paretian optimum with the conditions characterizing perfect competition, and, after restating the conditions for an optimum in terms of costs, to examine briefly the marginal cost pricing principle as a policy guide for the attainment of a Paretian optimum.

CHAPTER II

THE PRACTICAL APPLICATION OF THE PARETIAN CONDITIONS

In the derivation of the marginal conditions discussed in Chapter 1, the usual constraint assumed operative in the maximization of welfare was that of the transformation function. With regard to the practical problem of maximizing welfare there are, however, many other additional constraints[1] that must be taken into consideration. Such constraints have the force of causing a divergence between the marginal conditions, any may take the form of the existence of external economies and diseconomies, the existence of governmental or other institutional interventions in the product or factor market, or the existence of taxes and subsidies.

A divergence in the product market between marginal private values and marginal private costs would be caused by the monopolistic element of one seller of a particular product or by the monopsonistic element of one buyer of a particular product. Similarly, in the factor market there

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[1]Otto Eckstein in "A Survey of the Theory of Public Expenditure Criteria," in Public Finances: Needs, Sources, and Utilization (National Bureau of Economic Research, Princeton: Princeton University Press, 1961), pp. 450-2, classifies the various constraints operative in any public expenditure model as physical, legal, administrative, distributional, political and financial constraints.

might exist a monopoly element on the part of, say, labour as sellers of their work or there might exist monopsony elements on the part of employers who purchase labour or some other factor of production.

A divergence between marginal social net product and marginal cost may result from the existence of any one of six categories of[2] external economies and diseconomies. There may be an external economy associated with production, as in the afforestation program of a timber company; with employment, as in the employment of highly educated men; or with consumption, as in the wearing of well designed clothes. On the other hand, there may be an external diseconomy associated with production, as in the stench permeating a community which surrounds a distillery; with employment, as in the use of coal, which produces smoke, instead of the use of electricity; or with consumption, as in the driving of a noisy automobile.

The several marginal conditions may also diverge, (not as between different conditions but rather as between situations calling for the satisfaction of a particular condition), as a result of the existence of institutional regulations. The government may impose direct controls in the factor market on the allocation of factors of production and on the price of certain factors of production, such as the setting of maximum hours of work or

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[2] J. E. Meade, Trade and Welfare (London: Oxford University Press, 1955), pp. 18-21.

minimum wage rates. In the product market the government might, once again, institute a scheme for the rationing of goods or might set price controls as in the pricing of certain agricultural products. Quantitative institutional arrangements, mainly the result of attempts to overcome inconveniences and exhorbitant administration costs, also operate in such a way as to cause divergencies in the marginal conditions. Examples of such institutional constraints are those which determine the total amount of work that a man may do each day, such as the eight-hour day, and those which determine the type of work; it is not usually possible for a man to work half a day as a milkman, a quarter of the day as a gardener and the remaining quarter as a bricklayer.

Divergencies between the marginal value of the product and the marginal factor cost contained in that product may be caused by the existence of taxes and subsidies. Indirect taxation withholds from the producer a part of the price paid by the consumer and disturbs rates of substitution between the taxed good and the other goods; while direct taxation withholds from the receiver of income a part of the price paid for its efforts in producing the product and disturbs the rates of substitution between leisure and all other goods.

In the last few paragraphs some of the causes of divergencies between the various marginal conditions have

been noted. Such examination of these causes of divergence and, generally, of the constraints which prevent the simultaneous universal satisfaction of the marginal conditions has in the past led various writers to the conclusion that the conditions of perfect competition were compatible with those which defined a Paretian optimum. As mentioned in Chapter 1,[3] Pareto felt that free competition led to maximum satisfaction. In a similar manner, Barone[4] held the conditions characterizing a maximum of free competition to be exactly the same as those characterizing a position of maximum collective welfare. Such recognition of the compatibility of these two sets of conditions has, accordingly, resulted in the constant recurrence of the theme which expounds the goodness of a perfectly competitive economic system. In particular, mention should again be made of A. Marshall's strong disposition in favor of free competition.[5] And indeed, an examination of the several conditions defining a Paretian optimum yields the conclusion that a régime of perfect competition would give rise to the attainment of a Paretian optimum.

In the discussion that follows, "perfect" competition will be distinguished from "pure"competition. "Pure"

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[3]Above, p. 4.

[4]Above, p. 5.

[5]Above, pp. 6-7.

competition requires the fulfillment of two conditions. First, "there must be a large number of buyers and sellers so that the influence of any one or of several in combination is negligible."[6] And, secondly, "goods must be perfectly homogeneous"[7] and divisible. Taken together, these two conditions have the force of ensuring that no buyer or seller has control over supply and price. To these conditions must be added two further conditions in order that perfect competition be attainable. Namely, there must be "an absence of friction in the sense of an ideal fluidity or mobility of factors such that adjustments to changing conditions which actually involve time are accomplished instantaneously in theory," thereby, implying zero transportation costs, and there must be "perfect knowledge of the future and the consequent absence of uncertainty"[8] on the part of every consumer and entrepreneur. With this definition of perfect competition, the marginal conditions of a Paretian optimum will be examined separately in order to show that they are all satisfied under the conditions of perfect competition.

The first marginal condition, referring to the

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[6] E. H. Chamberlin, The Theory of Monopolistic Competition (6th ed.; Cambridge: Harvard University Press, 1948), p. 7.

[7] Ibid.

[8] Ibid., p. 6.

marginal rate of substitution between any two products being common to every individual who consumes both goods, is satisfied under a régime of perfect competition as each consumer of any one product faces the same market price for that commodity as any other consumer. And, according to the postulated rational behavior of the consumer expressed in terms of his attempt to maximize satisfaction, each consumer will equate his marginal rate of substitution between any two products to the ratio of their prices. The marginal rate of substitution between any two products is thus made equal to the price ratio of these two products and, as these price ratios are everywhere the same, the marginal rate of substitution between any two products is held in common by all consumers of the two products.

In a similar manner, the second marginal condition can be shown to be satisfied under a regime of perfect competition. Assuming rationality of behavior of producers as expressed in the attempt to maximize profits, the marginal rate of transformation between any two products will be equated to the ratio of the prices of these two products by each producing unit. Because each producer under the conditions of perfect competition receives for a particular product the same price as any other producer, the price ratio between any two products will be the same for all producers, and, accordingly, the marginal rate of transformation between any two products will be the same for all producers of these two products.

The third marginal condition can, in a completely analogous manner, be shown to be satisfied under conditions of perfect competition if a factor of production is treated as a negative product. In effect, the producers of any one product will face the same prices for the factors which produce the product and will likewise be confronted with the same selling price of the product. Because each producer, in his attempt to maximize profits, will equate the marginal rate of transformation between a factor and the product it produces to the price ratio of the prices of the factor and the product, and because the prices are the same for all producers, throughout firms which produce the product using that particular factor the marginal rate of transformation between the factor and the product will be equated to the ratio of their prices.

The satisfaction of the fourth marginal condition under a régime of perfect competition can also be shown in the above manner. In his attempt to maximize profits, each producer will equate the marginal rate of technical substitution between any two factors to the ratio of the prices of these two factors. But because the same price for any particular factor confronts all producers, the ratio of the prices of any two factors will also be the same for all producers and, accordingly, throughout the economy, the marginal rate of technical substitution

between any two factors will be equated to the price ratio of these two factors by all producers who employ these two factors.

In order to show that the fifth marginal condition is satisfied under the conditions of perfect competition it is necessary to combine two of the proceeding proofs, those for the marginal conditions one and two. It was shown that under perfect competition, the marginal rate of substitution between any two products is equated by all consumers of these two products to the ratio of the prices of these products. It was then shown that the ratio of the prices of any two products was, under the conditions of perfect competition equated by each producer of these two products to the marginal rate of transformation between these two products. Accordingly, the marginal rate of substitution between any two products is made equal to the marginal rate of transformation between these same products under the conditions of perfect competition.

The sixth marginal condition relating to the payment to each factor of production an amount equivalent to the value of its marginal physical product is also satisfied under the conditions of perfect competition. Under the assumption of profit maximization, the various factors of production will be employed up to the point at which the value of the marginal physical product of a unit of labour is equal to the unit price of that factor. Thereby, the

value of the marginal physical product is made equal to the marginal rate of transformation of the factor unit's time into the product. Similarly, in his attempt to maximize satisfaction each factor owner will equate the marginal rate of substitution between the renumeration received for the use of the factor and the utility of leisure or of the value of work done for himself, to the rate of reward which the factor could command. But under the conditions of perfect competition the rate of reward commanded by a factor is made equivalent to the value of the marginal physical product. Accordingly, the marginal rate of substitution between the renumeration for the services rendered and the value of the services that could have been rendered to the factor owner by use of the factor himself is made equivalent to the marginal rate of transformation between the amount of time spent by the factor in the production of the product and the amount of the product produced.

In maximizing profits, the individual or firm will attempt to equate the marginal rate of substitution between the holding of an asset at one period of time and the holding of the same asset at a different period with the ratio of the prices of the asset at the different periods of time. The common interest rate confronting all individuals and firms will, thus, guarantee that marginal rate of substitution between resource control at any two periods of time will be the same for every individual or firm.

In general, perfect competition in the consumers' sector implies that the consumer equates his marginal rate of substitution between any two commodities with the ratio of their prices; and, since prices are the same for all consumers, the marginal rates of substitution between any two products are the same for all consumers. In the producing sector, perfect competition implies that the price of a commodity is not altered by variations in the levels of output of various firms and that the prices of inputs are not altered by variations in the level of purchases of individual firms. If profit is to be maximized, the marginal rates of technical substitution between any two factors must be equal to the ratio of their prices; and, since each firm pays the same price for its inputs, the marginal rate of substitution between any two factors must be the same for all firms. Similarly, perfect competition results in the necessary equivalence of the marginal rate of product transformation between any two products to the ratio of their prices among all the firms producing the two products and the necessary equivalence of the marginal rate of transformation of an input into an output to the ratio of the prices of the input and output among all firms using that input to produce that output.

The above discussion, then, yields the conclusion that a position of maximum welfare is compatible with the conditions

of perfect competition.[9] Accordingly, the question of whether or not a position of maximum welfare exists can be posed as the question of whether or not perfect competition exists. Given that certain conditions of perfect competition; those of perfect knowledge on the part of all consumers and producers, of perfect divisibility of all factors and products, and of perfect mobility in the sense of zero costs of transportation, can not possibly be achieved, this question is further reduced to that of whether or not a régime of pure competition exists.[10]

In order for pure competition to exist it is necessary that no buyer or seller has control of the price of any factor or product, that is, the economic unit, be it a consuming or producing unit, must be small relative to the overall size of the market. The existence of pure competition, then, is characterized by the two conditions put forward by Pareto and Barone,[11] namely that costs of production in a technical sense be a minimum and that marginal costs equal price. The satisfaction of these

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[9]A. P. Lerner in Economics of Control (New York: The Macmillan Co., 1944), pp. 76-7, shows the satisfaction of his five welfare equations under the conditions of perfect competition.

[10]It is important to distinguish between the continuous attainment of a welfare optimum under a régime of perfect competition and a continuous tendency towards a welfare optimum as would result under a régime of pure competition. In practice, one is concerned with a constant tendency towards a welfare optimum.

[11]Above, p. 4.

two conditions has, thus, become the objective of those persons interested in the maximization of social welfare in an economy characterized by indivisibilities and monopolies.[12] For in order that a Paretian optimum be achieved, it is necessary that production costs be a minimum and that price equals cost. Indeed, these two conditions may be thought of as representing a restatement of the optimum conditions in terms of costs;[13] by which restatement the policy implications of the marginal conditions are more easily understood.

The first criterion for the attainment of a social optimum can be thought of as the minimization of costs of production in a technical sense. This condition, however, is not of primary concern to the policy maker. Rather it is considered a problem of maximization of a lower level optimum.[14] Like the assumption that the consumer in maximizing his satisfactions will equate his marginal rate of substitution between any two products to the ratio of the prices of these products, so also business firms may be assumed to minimize their costs of production

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[12] K. E. Boulding, "Welfare Economics," in A Survey of Contemporary Economics, Vol. II, ed. B. F. Haley, (New York: Blakiston, 1952), p. 25.

[13] A. Bergson, "Socialist Economics," in A Survey of Contemporary Economics, Vol. I, ed. H. S. Ellis, (Philadelphia: Blakiston, 1948), p. 424.

[14] E. J. Mishan, "Second Thoughts on Second Best," Oxford Economic Papers, Vol. 14 (October, 1962), pp. 212-3.

given the underlying assumption that profits are to be maximized. The assumption of producer rationality thus implies that production is carried on in a manner such as to result in minimum costs. Hence, it is with the second condition, requiring the equivalence of prices to costs, that the policy maker is primarily concerned. According to Mishan, the policy maker is concerned with the achievement of top-level optima.[15]

The proposition that marginal cost should be equated to price was advanced by Alfred Marshall as a result of his analysis of consumers' and producers' surplus. His celebrated doctrine that taxes should be levied on the "production of goods which obey the law of diminishing return," and subsidies or bounties should be granted on the "production of those goods with regard to which the law of increasing return acts sharply,"[16] developed from his belief that an optimum was attainable only if marginal cost was equal to price. Elaborating on this proposition was the analysis of Pigou who boldly expounded the thesis that marginal cost must equal price in order that a social optimum be achieved.[17]

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[15] Ibid., p. 213.

[16] A. Marshall, Principles of Economics (8th ed. London; Macmillan and Co. Ltd., 1930), pp. 472-3.

[17] A. C. Pigou, Economics of Welfare (4th ed. London: Macmillan and Co. Ltd., 1932), pp. 789-810. See above, p. 8, for Pigou's statement regarding the equality of marginal cost and price as a necessary condition for ideal output.

Although the fundamental proposition that price must equal marginal cost stems directly from the work of Marshall and Pigou, it is the work of A. P. Lerner[18] which has popularized this concept and which has defined it with precision. Indeed, this fundamental proposition is today referred to as the Lerner Rule.

According to Lerner, there are three steps which are necessary if the problem of the optimum allocation of factors is to be solved. The first step requires "a free market in the sale of the consumption goods so that there can be established an optimum allocation of whatever goods are produced."[19] The second also utilizes the price mechanism and requires "a free market in the sale of the factors of production to the managers of production so that the price of any factor, payable by the manager who acquires it for use in the factory, is the same as the price paid by any other manager."[20] Representing the third step, then, in the optimization of the allocation of resources is Lerner's directive to every manager of production, as follows:

> If the value of the marginal (physical) product of any factor is greater than the price of the factor, increase output. If it is less, decrease output. If it is equal to the price of the factor continue producing at the same rate. (For then the right output has been reached.)[21]

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[18] Lerner, Economics of Control.

[19] Ibid., p. 63.

[20] Ibid.

[21] Ibid., p. 64.

If perfect competition in the factor market is assumed so that marginal cost is equal to the value of the marginal physical product of the factor, then, the rule may be easily restated in terms of marginal costs.[22] In the words of Hotelling, "the criterion for a small increase in production is still that its cost shall not exceed what buyers are willing to pay for it; that is, the general welfare is promoted by offering it for sale at its marginal cost."[23]

The formulation of this marginal cost pricing rule as a necessary condition for the attainment of an optimum position has not been accomplished without confusion and controversy. Especially in the discussion of marginal cost as opposed to average cost has controversy developed.[24] In the case of constant costs, average costs are equal to marginal costs so that there is no problem. However, the various cases of fixed factors and indivisibilities which characterize the real world do not give rise to production at constant costs; accordingly, marginal costs and average

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[22] Ibid., p. 98-9.

[23] H. Hotelling, "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates," Econometrica, Vol. 6 (July, 1938), pp. 255-6.

[24] The best summary of the marginal cost controversy would appear to be that of Nancy Ruggles, "The Welfare Basis of the Marginal Cost Pricing Principle," Review of Economic Studies, Vol. 17 (1949-50), pp. 29-46; and her other article, "Recent Developments in the Theory of Marginal Cost Pricing," Review of Economic Studies, Vol. 17 (1949-50), pp. 107-26.

costs differ. Nevertheless, the fundamental proposition that marginal cost must equal price for the attainment of the optimum holds "regardless of the relation of marginal and average costs, regardless of whether price is above average cost and there are 'profits' (as might be so in the case of 'fixed factors') or below average cost and there are losses (as might be so also in the case of 'fixed factors,' and very likely would be so in the case of large indivisibilities)."[25]

Another point of controversy that has developed over the condition requiring the equivalence of marginal cost to price for the attainment of an optimum, is the question of whether or not this condition would be equally well satisfied by the proportionality of price to marginal cost in a ratio that prevailed everywhere the same throughout the economy. Indeed, along with the formulation of the marginal cost pricing rule there develops also the implicit assumption that if marginal cost cannot be exactly equated to price but yet can be set in some universally common ratio that the optimum condition will still be satisfied. In Pigou's work there is the suggestion that, as long as all departures from the equality of marginal cost to price were of the same direction and magnitude, the ideal output would

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[25]Bergson, loc. cit., p. 425.

attain. Accordingly, he states that "if in all industries the values of the marginal social and marginal private net products differed to exactly the same extent the optimum distribution of resources would always be attained, and there would be, on these lines, no case for fiscal interference."[26]

This belief that equal deviation throughout the economy was consistent with ideal output was taken up from Pigou, and elaborated on, by R. F. Kahn in his article, "Some Notes on Ideal Output,"[27] in which he states that "if the ratio of the value of the marginal social product of each factor to the value of its marginal social product is everywhere the same, the output of every industry is ideal."[28] This statement of Kahn may be referred to as the Proportionality Thesis.[29] Perhaps a more useful statement of this thesis is that given by A. M. Henderson, namely, that "the ideal distribution of resources between industries will be obtained if the ratio of prices to marginal cost is the same in all industries."[30] The

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[26]Pigou, loc. cit., p. 224.

[27]R. F. Kahn, "Some Notes on the Ideal Output," Economic Journal, Vol. 45 (1935) pp. 1-35.

[28]Ibid., p. 11.

[29]The phrases "Proportionality Rule" and Proportionality Thesis" are coined by L. MacKenzie in "Ideal Output and the Interdependence of Firms," Economic Journal, Vol. 61 (December, 1951), pp. 785-6.

[30]A. M. Henderson, "The Pricing of Public Utility Undertakings," The Manchester School, (September, 1947), p. 242; as quoted by MacKenzie, loc. cit., p. 785.

accompanying Proportionality Rule, derived from this thesis, stated in the words of Kahn that "it is socially desirable to expand those industries in which competition is more imperfect than the industries with which they compete for their factors of production and to contract those in which the opposite prevails."[31] Or more specifically, if all industries are arranged in order of their degree of imperfection, "it is socially desirable to expand those industries which lie above the "average" industry and to contract those which lie below."[32]

Now, if the problem of allocating a fixed supply of resources is conceived of as being restricted to allocation among fully integrated firms whose only transactions are those with individuals acting in the capacity of consumers or owners of resources, then, the Proportionality Thesis holds and the application of the Proportionality Rule would bring about the optimum allocation of resources.[33] However, in the world today it is not realistic to assume that final products are produced by completely integrated firms. Rather, it is more expedient to conceive the process of production as taking place in several stages with firms selling intermediate products to each other as well as

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[31] Ibid., p. 21.

[32] Ibid., p. 21-2.

[33] MacKenzie, loc. cit., pp. 788-9.

selling final goods to consumers. It is the problem of the accumulating effects of a divergence which occurs at an early stage of production between marginal cost and price that has led to the disproof of the Proportionality Thesis.

In his attempt to measure the divergence between the price of the product and its marginal social cost, in cases of monopoly, A. P. Lerner had recognized the accumulating effects of monopoly power that existed throughout the several stages of the production process of a particular final product. He outlined his example as follows:

> If in any stage there is a divergence, price being above marginal cost, that divergence is a gap in the social cost. The social cost can then be calculated by multiplying the price by a factor for each stage in production, each factor being the ratio of the marginal cost to the price in the corresponding stage. Thus, if there are five stages and in each stage the degree of monopoly was 1/5, marginal cost over price in each stage is 4/5, the social cost is $(4/5)^5$ of the price of the final product. and by our formula the social degree of monopoly is $1 - (4/5)^5$.[34]

But Lerner did not at that time use this observation to discuss the validity of the Proportionality Thesis. However, in his book, Economics of Control,[35] Lerner does point out the defectiveness of the Proportionality Thesis. Using the example of the allocation by an individual of his time between work and leisure, Lerner concludes that "it is not possible

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[34] A. P. Lerner, "The Concept of Monopoly and the Measurement of Monopoly Power," Review of Economic Studies, Vol. 1 (June, 1934), p. 172.

[35] Lerner, Economics of Control, pp. 100-5.

for the ratio between vmp and pf (or between p and vmf or between vmp and vmf) to be the same in all uses of all factors unless this proportion is unity."[36]

The inadequacy of the Proportionality Thesis is also pointed out by Reder. Assuming that direct services of factors may be adequately treated as products, Reder demonstrates that marginal cost must equal price if the ratio of marginal cost to price is the same for all products:

> The answer is yes, if we include direct services of factors as products; but if the ratio of the price of every product (including direct services) to its marginal cost is the same, then the marginal cost of producing each product must be equal to its price. For the ratio of price to marginal cost of any product, X, in terms of any other product, Y, is the reciprocal of that ratio of Y in terms of X. The only value of the two ratios which makes them equal is unity; i.e. the only possible ratio of price to marginal cost which could hold for all products simultaneously is unity. Therefore, if the ratio of price to marginal cost is the same for all products, price must equal marginal cost for each of them.[37]

Reder then proceeds to show that, if direct services of factors are not conceived of as products, the equivalence of the ratio of price to marginal cost of all products does not imply the attainment of an optimum allocation of resources; for, if this ratio is not equal to unity, the sixth marginal condition will be violated.[38]

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[36] Ibid., p. 102.

[37] M. W. Reder, Studies in the Theories of Welfare Economics (New York: Columbia University Press, 1947), p. 42, n. 5.

[38] Ibid.

Finally, MacKenzie gives a thorough analysis of the Proportionality Thesis and Rule. After dividing the problem of the allocation of resources into the distinct categories of "allocation among lines of production, among the industries in successive stages of a given line of production and among the firms of a single industry, MacKenzie examines the first two of these three problems of allocation and shows that the application of the Proportionality Rule would lead immediately to a divergence between the prices to consumers and the marginal rates of substitution among final products and to a divergence between the prices to firms and the marginal rates of substitution between factors of production, in particular, between ultimate factors and intermediate products.[39] Essential to his proof of the defectiveness of the Proportionality Thesis is the notion of the "accumulation of the divergence between the price of the productive service and the value of the marginal product at later stages"[40] and also the idea of inflation of the price of the intermediate product over the cost of producing additional units of it to the extent that "it is always possible that some group of productive services in the later stage could produce in the earlier stage more of some type of intermediate product than is required

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[39]MacKenzie, loc. cit., p. 790.

[40]Ibid., p. 791.

to restore the level of output in the latter stage."[41]

The results of the analyses of Lerner, Reder and MacKenzie, thus, yield the conclusion that the Proportionality Thesis is invalid; that is, that the equivalence of the ratio of marginal cost to price for all products does not define a position of maximum welfare. Little concludes his discussion of the problem by stating that, as a theoretical proposition, the case is "overwhelming against the proportionality thesis."[42] In sum, the attainment of a position of maximum welfare requires the equality of marginal cost to price for each and every product; and such a condition is satisfied by the existence of perfect competition. Accordingly, perfect competition is a sufficient, although not necessary, condition for the attainment of a Paretian optimum.

The conditions of perfect competition are not, however, ubiquitously existent throughout the real world. The various constraints discussed at the beginning of this chapter do, in fact, characterize the real world and effectively prevent the simultaneous, universal satisfaction of the several marginal conditions which define a Paretian optimum. Accordingly, welfare must be maximized subject not only to the usual constraint imposed

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[41] Ibid.

[42] I. M. D. Little, A Critique of Welfare Economics (2nd ed.; Oxford: Clarendon Press, 1957), p. 164.

by the transformation function but also to the several possible additional constraints which cannot be removed.

Given the existence of an imperfectly competitive economy, some of whose imperfections can be removed while others cannot, that is, given the fact that certain of the Paretian optimum conditions are capable of satisfaction while others are not, the problem now faced is that of determining the proper criteria by which various policies can be evaluated according to whether they constitute a movement towards or away from a position of maximum welfare. Is it correct to assume that the Paretian optimum conditions are appropriate as policy guides in the sense that a situation in which more of these conditions are satisfied is superior to one in which fewer are satisfied? Although the Proportionality Thesis has been proved defective as a description of optimum position, can the Proportionality Rule serve as a policy guide in the sense that a situation in which all the Paretian marginal conditions diverge to the same extent is superior to one in which the divergences vary in direction and magnitude?

In the past these two questions have been answered in the affirmative. With the development of, and the many refinements made to, the marginal conditions defining a Paretian optimum there is the implicit and sometimes explicit belief that, if the Paretian conditions were not all universally satisfied, the optimum would be approached

by a closer approximation to the equality of the several marginal conditions. When talking about the two fundamental conditions that characterize a collective maximum, Barone states that "the maximum is more nearly attained the more perfectly they are realized."[43] Also typical is the conclusion of Pigou to the effect that, "when complete equality among the values of marginal social net product is wanting, a diminution in the degree of inequality that exists among them is likely to benefit the national dividend."[44] In a similar manner, the belief in the adequacy of the Proportionality Rule as a guide to policy, is held either implicitly or explicitly by those who developed the Proportionality Thesis.

In the more recent past, however, especially in the field of international trade and the discriminatory reduction of tariffs, doubts have been raised concerning the hitherto unquestioned supremacy of the Paretian optimum conditions as sufficient conditions for a movement towards the optimum. The several attempts of economists in various fields of economics to set out the conditions which would define a maximum position,

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[43]Barone, loc. cit., p. 289.

[44]Pigou, loc. cit., p. 137. Unlike Barone who felt that a diminution in the degree of inequality was certain to lead to an increase in welfare, Pigou only felt that it was likely to lead to an increase.

given the preclusion of a Paretian optimum because of the impossibility of eliminating certain divergences and constraints, will be surveyed in the next chapter.

CHAPTER III

OPTIMA IN THE PRESENCE OF ADDITIONAL CONSTRAINTS

In the foregoing discussion[1] of the Proportionality Thesis and of the attempt to achieve the ideal allocation of resources by means of the Proportionality Rule, mention was made of the analysis of L. W. MacKenzie. He had concluded that the structure of production would remain undistorted only if productive services were shifted among the various stages of production until marginal cost was everywhere equal to price. Subject to the condition that all firms supplying raw materials and intermediate products were perfectly competitive, equating price to marginal cost, and that, accordingly, those firms producing final goods could be conceived of as being fully integrated, the ideal allocation of factors of production was also to be attainable when the ratio of marginal cost to price was the same throughout the final-goods producing industries. MacKenzie then set out to examine the best allocation of resources given the condition that marginal cost could not everywhere be made equal to price; that is that a Paretian optimum was not attainable. In particular, he

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[1]Above, pp. 38-46.

set out to determine whether or not a movement towards the best distribution occurs when as many of the divergencies between marginal cost and price as possible are eliminated.

As before, MacKenzie's model describes the economy as being composed of industries producing both intermediate and final products with certain industries such as transportation, power and communication selling their products both in the form of final and intermediate goods. Defining "degrees of inflation" as the ratios between actual and competitive prices[2] where competitive prices are defined as those consistent with the equality of marginal cost to price throughout the economy, MacKenzie shows that, where production is not characterized by fixed proportions, "innumerable patterns of factor movements would be able to effect substitutions between given outputs, and what is worse, some of these ways may be far superior to others, though they will not be chosen in a free market."[3] In effect, MacKenzie suggests that a system, in which marginal costs differ from prices in varying degrees, depending on the efficiency of production and the exact position of a particular stage in the whole production process, will be the best for attaining an undistorted allocation of resources given that certain marginal cost and price

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[2]L. W. MacKenzie, "Ideal Output and Interdependence of Firms," Economic Journal, Vol. 61 (December, 1951), p. 795.

[3]Ibid., p. 798.

combinations can not be everywhere brought into equality.[4]

A similar problem of maximization in the face of constraints in addition to those contained in the transformation function is that analyzed by A. Smithies in his article, "The Boundaries of the Production and Utility Function."[5] As a first approximation the firm is assumed to maximize its profits subject to the constraint of the production function by equating the value of the marginal products of various factors to the marginal costs of these factors. However, it is postulated by Smithies that the firm is faced with at least one constraint in addition to that of the production function. Smithies then goes on to show that the operation of this additional constraint within the maximizing problem results in a distortion of the original marginal equalities required for maximization and, specifically, that the marginal cost of the various factors may be above, below, or equal to the value of their marginal products as set out in the first maximizing problem.

Smithies sets out a practical example of the problem he envisages as follows:

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[4]R. G. Lipsey and K. Lancaster, "The General Theory of Second Best," Review of Economic Studies, Vol. 24 (1956), pp. 16-7.

[5]A. Smithies, "The Boundaries of the Production and Utility Function," in Explorations in Economics (London: McGraw-Hill, 1936), pp. 226-35.

> Suppose the independent variables of the production function of a pig-iron producer are labor, iron-ore, coke, and blast furnace capacity, then in order to produce a given amount of pig iron per unit of time there is an irreducible minimum of iron ore greater than zero necessary, and this is independent of the amounts of other factors used.[6]

Using mathematical symbols, Smithies sets out a simple production function as a function of three variables x, y and z. One of these variables, say x, is, then, assumed to be required in certain minimum amounts dependent on the level of total output alone. This assumption has the force of imposing a boundary constraint on employment of the factor x once the profit maximizing output level has been specified.

The firm seeks to maximize revenue by equating the value of the marginal productivity of the various factors to the marginal cost of these various factors. These particular marginal equalities are derived from the maximization defined as the difference between the value function and the total cost function. An examination of the particular equations for the attainment of this firm's profit maximum position yields certain values of the factors x, y and z. It is, then, postulated that the amount of the

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[6] Ibid., p. 326. The description of this constraint, as set out by Smithies, is somewhat misleading. It is here interpreted to mean that in order to produce 100 tons of pig-iron it is necessary to use at least 120 tons of iron ore. Employment of iron ore at levels in excess of 120 tons for the production of the same 100 tons of pig-iron is dependent on the amounts of the other factors used. That is, the constraint is inoperative above the input-level of 120 tons of iron ore.

variable factor x, required to satisfy the marginal equations, derived from the simple production function, is an amount less than that which is technically needed. That is, in the example of the pig iron producer, the number of tons of iron ore that the satisfaction of the marginal conditions required is less than the number of tons of iron ore that is needed to produce the desired level of pig iron; this desired level is the one specified as yielding maximum profits given existing product and factor market prices. Accordingly, to produce that level of output deemed to yield a maximum level of profit, it is necessary to employ units of the factor x beyond the point at which the marginal cost of x is equal to the value of its marginal productivity. With production being carried on at the x boundary the function[7] defining the boundary condition of x will then be imposed on the other factors.

Revenue is now maximized with the aid of a Lagrange multiplier subject to the original constraint of the cost function and also subject to the additional constraint in the form of the boundary condition for x resulting from the violation of this boundary condition by the initial maximizing process. The marginal equalities, defining a position of maximum revenue, given the additional constraint, are now much more complicated than the initial marginal equalities.

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[7]Smithies, ibid., p. 327, explains that this additional constraint can not be taken care of by discontinuities in the production function without using highly complex parameters.

Since the boundary condition for x has been violated, an unconstrained or "free"[8] maximum is impossible. Indeed, the factor x is being employed to the extent that the value of its marginal product is less than its marginal cost. Employment of the other factors now takes place according to a complex set of marginal equations dependent on the derivatives of the function expressing the certain irreducible amount of x that it was found necessary to employ; the other factors may be employed beyond the point at which their marginal cost equals the value of their marginal products, in terms of the original marginal equalities of the free maximum; at the point of equality; or, at a level indicating that marginal cost is below the value of the marginal product.[9]

Alternately, the firm may be conceived of as maximizing profits according to its ability to satisfy a new and more complex set of marginal equalities. The essential point is that the firm can no longer rely on the original set of marginal equalities derived from the initial attempt to attain and define a free maximum. Given the additional constraint, the firm is faced with a new set of marginal equalities which must necessarily be satisfied if revenue is to be maximized. Furthermore, in this new set of marginal

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[8] Ibid., p. 330. This "free" maximum is free in the sense that the additional constraint has not been considered even though, strictly speaking, it should have been included in the original complete model.

[9] Ibid.

conditions "there is no a priori reason for thinking that the nature of the inequality will be the same for all factors,"[10] that is, the divergence between the original marginal equality and the new marginal equality for each factor will not necessarily be of the same magnitude nor of the same direction for all factors.

This attempt to find a maximum solution subject to an additional constraint is essentially a maximization problem of a lower-level optimum. For, presumably, the firm in its attempt to minimize costs and, thereby, to maximize profits will satisfy the new complex set of marginal equalities resulting from the operation of the additional constraint.

In the same manner, the exchange optimum, which requires the equality of marginal rates of substitution between all pairs of goods for all consumers purchasing these two goods, may be thought of as a lower-level optimum. As a first approximation, an individual consumer may be assumed to maximize a utility function, whose shape is determined by the individual's tastes, subject to the usual constraints of his budget and the prices of all the various goods he desires. According to Smithies, however, a distinction between tastes as opposed to habits can be made. The habits of an individual, then, as opposed to his tastes, may be assumed to prevent the attainment of maximum satisfaction,[11]

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[10] Lipsey and Lancaster, loc. cit., p. 16.

[11] Smithies, loc. cit., p. 331.

and may be conceived of as additional constraints in the maximization problem. They may take the form of absolute maximum or minimum amounts of certain commodities. The example given by Smithies is that of two Englishmen transported to America:

> To take a concrete example, suppose an Englishman is transported to America, and the traditional prejudices of his race prevent him from adjusting his pattern of consumption to American conditions until he has suffered for some time. However, he is eventually convinced that there will be no dire consequences if he overthrows his prejudices. But when he returns to England, he immediately reverts to his old way of living. Now suppose a second Englishman who lives in England in exactly the same way as the first, but who is not so bound by tradition. When he comes to America, he immediately adjusts himself to American conditions and when he returns he reverts to his old consumption pattern.[12]

According to Smithies the first Englishman's traditional habits may be thought of as placing a rigid constraint[13] on the maximization of his utility function at least in the short run. The individual's consumption of a particular good may be subject to either an absolute maximum or absolute minimum boundary condition or it may be dependent on the quantities of other goods consumed;[14] in any case, the

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[12] Ibid., pp. 331-2.

[13] This constraint is only additional in the sense that it was not included in the original utility function which was defined simply as a function of the individual's consumption of various commodities. Strictly speaking, these habits, formed by tradition, should be a part of the individual's utility function but would require the use of highly complex parameters.

[14] Ibid., pp. 332-3.

individual may be conceived of as maximizing his utility function subject to an additional constraint. Accordingly, as in the case of the maximization of revenue and the achievement of a production optimum subject to an additional constraint, so also the achievement of an exchange optimum subject to an additional constraint gives rise to a new set of marginal equalities each of which may once again differ in magnitude and direction from the original set of marginal equalities, all of which equate the marginal utility of every commodity to the marginal dissatisfaction of acquiring it,[15] and are derived from the "free" maximization of utility.

In a sense, then, the achievement of both a production optimum and an exchange optimum, subject to the existence of constraints in addition to those usually assumed operative, may be thought of as the achievement of a new Paretian production and exchange optimum respectively. And it should be noted that the new Paretian optimum in these two cases is defined by a set of marginal equalities which are generally more complex than those defining a "free" maximum and each of which may vary in the magnitude and direction of its divergence from the original marginal equality.

The value of these two examples of Smithies lies not in their illustration of the difficulty of attaining a second-best optimum given the preclusion of a Paretian

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[15] Ibid., pp. 333.

optimum but rather in their illustration of the difficulty of attaining a Paretian optimum given the operation of constraints in addition to those usually assumed operative in a first approximation of the maximizing problem. It is granted that these two examples of the maximization of a lower-level optima do not pose any problem to the policy maker; for, the individual and firm may be thought to maximize utility and profits, respectively, in accordance with the constraints assumed operative, and, accordingly, marginal rates of substitution and marginal rates of transformation are equated to price ratios. These examples do, however, show that the consideration of additional constraints necessitates the satisfaction of a new and more complex set of marginal equalities in order that an optimum position be attained.

Similar conclusions to those noted above in the problem of maximizing production and utility optima subject to additional constraints are reached in another field of economics when the achievement of an optimum position of exchange is sought subject to the additional constraint of taxation. Attention is now directed to the treatment by I. M. Little in his article, "Direct Versus Indirect Taxation,"[16] of the problem of attaining a position of maximum satisfaction subject to the constraint of any taxation scheme which effectively prevents the simultaneous

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[16] I. M. D. Little, "Direct Versus Indirect Taxation," Economic Journal, Vol. 61 (September, 1951), pp. 577-84.

satisfaction of all the Paretian optimum conditions.

The traditional model, with its certain extraordinary and misleading assumptions, especially that of the constant supply of labor, had long been used to prove conclusively the superiority of direct taxation over indirect taxation. Exemplifying this traditional theory was the work of H. Hotelling in his article, "The General Welfare in Relation to the Problems of Taxation and of Railway and Utility Rates."[17] Hotelling developed his "fundamental theorem" which stated that "if a person must pay a certain sum of money in taxes, his satisfaction will be greater if the levy is made directly on him as a fixed amount than if it is made through a system of excise taxes which he can to some extent avoid by rearranging his production and consumption."[18]

The traditional treatment[19] of the problem of taxation may be briefly summarized and illustrated with reference to the diagram of Figure II. Units of the commodity X are measured along the X axis and units of the commodity Y are measured along the Y axis; i_1i_1, i_2i_2

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[17] H. Hotelling, "The General Welfare in Relation to the Problems of Taxation and of Railway and Utility Rates," Econometrica, Vol. 6 (July, 1938), pp. 242-62 (242-69).

[18] Ibid., p. 252.

[19] This traditional treatment is outlined by Little, loc. cit., pp. 577-80, and also by R. A. Musgrave, The Theory of Public Finance (New York: McGraw-Hill Book Co., Inc., 1959), pp. 142-6.

and i_3i_3 represent part of the individual's indifference map for goods X and Y. The initial tax free position of equilibrium is given by the point E which indicates the tangency solution of the indifference curve i_3 with the given price line CB. An income tax is then levied; its amount is equal to CF of Y or DB of X. The new position of equilibrium is then given by the point L, the tangency solution of the indifference curve i_2 and the new price line FD which is parallel to the old price line CB. In order to raise the same amount of tax revenue through indirect taxation of the good X, that is an excise tax on the sales of X, the relevant price line would have to take the position CG, thereby giving the new equilibrium position at S on the indifference curve i_1. If the indifference curves are assumed to be continuously convex,[20] it is clearly seen that the indifference curve i_2 represents a higher level of utility than does the indifference curve i_1 and, accordingly, the position L is clearly seen to be superior to the position S. The Hotelling conclusion directly follows, namely, that tax revenue should be raised by an income tax rather than by an indirect tax on one good or on some goods.

By assuming that the supply of labor is not constant and, thereby, implicitly assuming that the consumption of leisure is not necessarily left unaffected by changes in the relative prices of X and Y, it can be shown that

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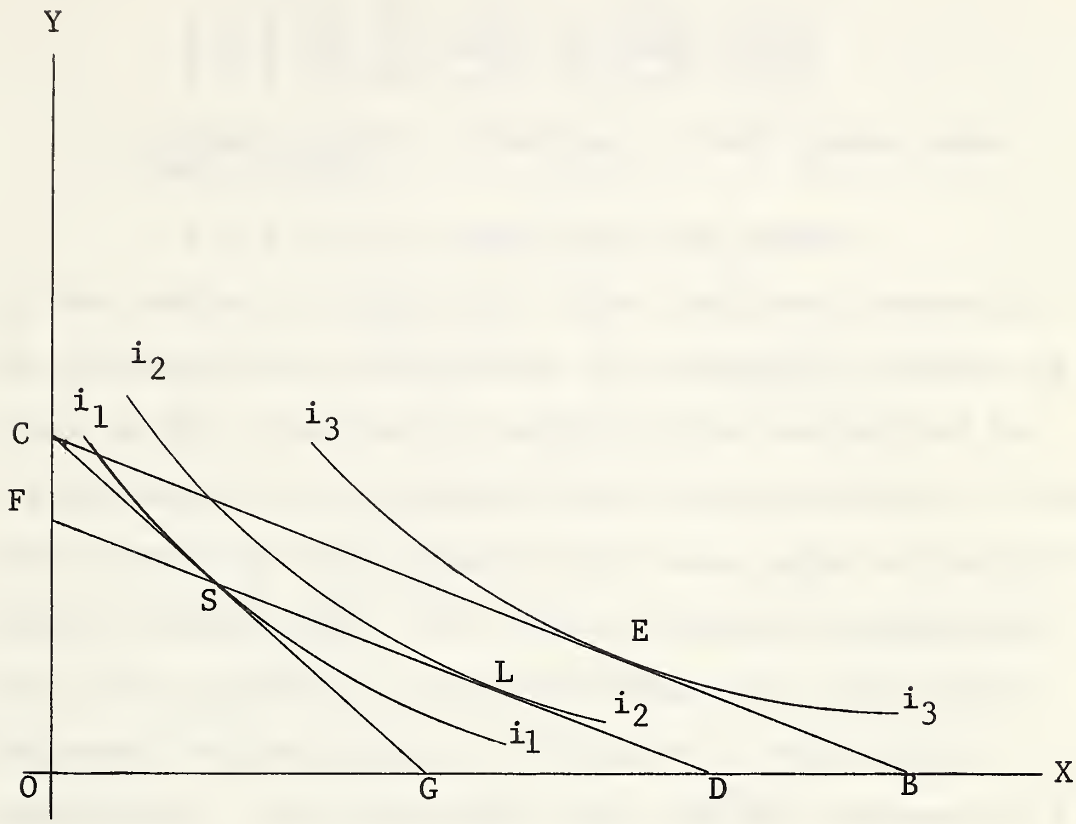
[20]Little, loc. cit., p. 577.

Figure II.--Traditional Indifference Curve Analysis of Direct Versus Indirect Taxation.



generalizations can not be made about the specific effects of indirect taxation as opposed to direct taxation. Assume a perfectly competitive economy in which three goods, X, Y and Z are consumed; one of which represents the good leisure. Let S stand for the marginal rate of substitution and let T stand for the marginal rate of transformation. Three general cases of taxation may, then, be outlined:

1. Direct Taxation:

$S = T$ for the pair of goods (X,Y)
$S \neq T$ for the pair of goods (X,Z)
$S \neq T$ for the pair of goods (Y,Z)

2. Indirect Taxation of one good other than leisure, say X:

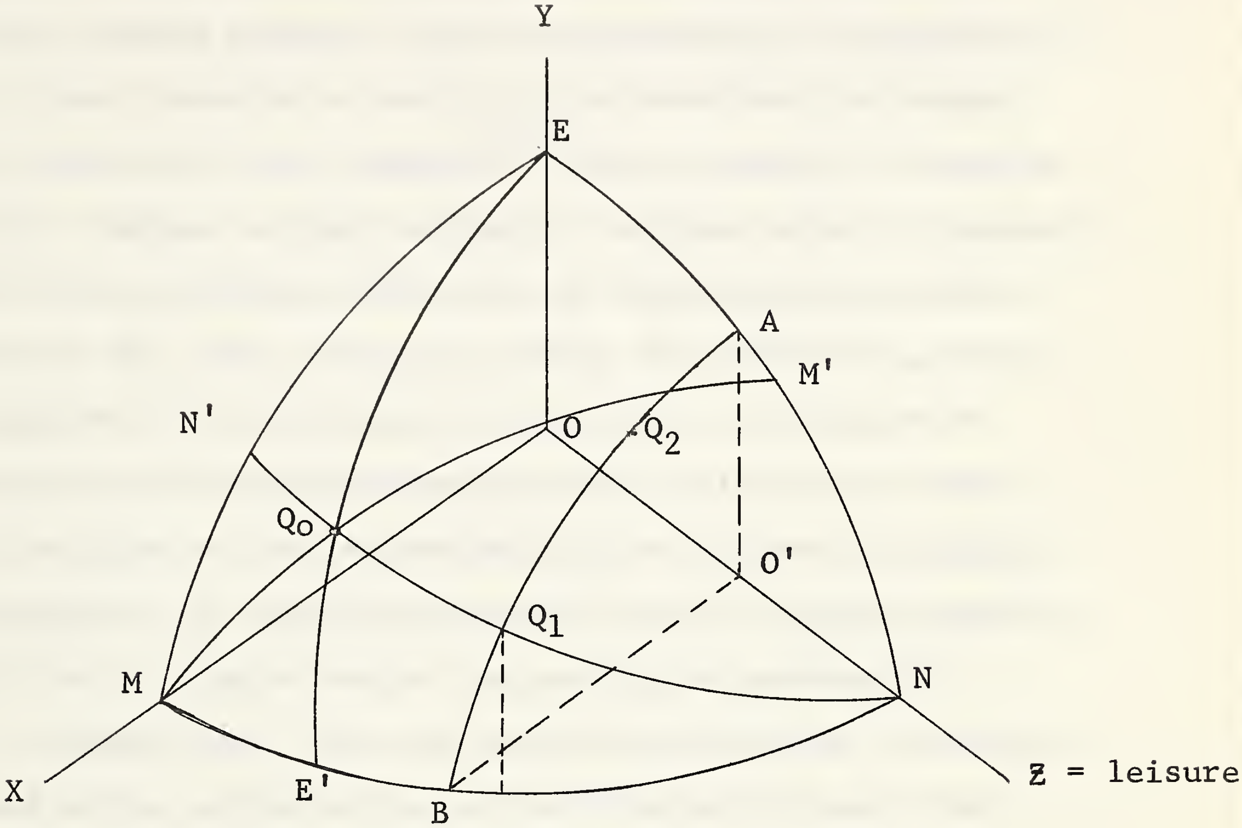
   $S = T$ for the pair of goods (Y,Z)
   $S \neq T$ for the pair of goods (X,Y)
   $S \neq T$ for the pair of goods (X,Z)

3. Unequal Indirect taxation of both goods other than leisure:

   $S \neq T$ for all three pairs of goods.

In the diagram of Figure III, when the fixed demands of the government are subtracted, the production surface is given as EMN. The budget surface is so manipulated by the government as to yield a direct correspondence of the point chosen by the community with some point on the production surface EMN. The x axis represents consumption of X and, in exactly the same manner, the y and z axes represent the consumption of the good Y and leisure, respectively. The lines EE^1, MM^1, and NN^1 represent all those points on the production surface for which, respectively, the partial rate of substitution of leisure for X equals the partial rate of transformation of leisure for X; the partial rate of substitution of leisure for Y equals the partial rate of transformation of leisure for Y; and, finally, the partial rate of substitution of X for Y equals the partial rate of transformation of X for Y. At the point Q_o, then, the conditions of the Paretian optimum are satisfied; for it is at this point that the marginal rates of substitution are equivalent to the marginal rates of transformation for the three goods.

Figure III.--Three-Commodity Model of Direct Versus Indirect Taxation.



If direct taxation is imposed and if it is assumed that the supply curve of labour is upward sloping, the resulting equilibrium position is given by some point on the line NN^1 lying between N and Q_o depending on the extent to which leisure is substituted for income. If, on the other hand, indirect taxes are levied against either X or Y, then, the equilibrium position will lie either on EE^1 between E^1 and Q_o or on MM^1 between M^1 and Q_o. And, finally, if Case III is assumed to hold with unequal rates of indirect tax being levied on X and Y, then, the equilibrium position

will not lie on any of the lines EE^1, NN^1 and MM^1. As a result of his analysis, Little reaches the conclusion "that nothing whatever can be said about the "superiority" of direct taxation unless it is certain that the amount of leisure will not change;"[21] for, no system of taxation which combines either varying or constant rates of indirect and direct taxation will move the equilibrium position back to Q_o. The only way in which the position Q_o can be regained is by the repeal of all taxes. It should be noted that the equilibrium position will be restricted to the area of the production surface defined by $Q_oM^1NE^1$, although it is entirely possible that the optimum position will be at a point below the production surface.[22]

Given, then, that the optimum position Q_o, characterized by the satisfaction of the Paretian optimum conditions is unattainable because of the existence of some scheme of taxation which prevents the satisfaction of one of the marginal equalities, Little concludes that there is no a priori way to judge between a system of direct taxation as opposed to a system of indirect taxation applied to one or some goods at either equal or unequal rates. He does, however, offer some tentative suggestions for deciding which taxes are best. In general, it will

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[21]Little, <u>loc. cit.</u>, p. 583.

[22]<u>Ibid.</u>

be best to tax those goods "for which the demand is least elastic."[23] Accordingly, where the demand for leisure is inelastic the income tax will be a good tax.

With the help of a mathematical model, these results of Little were then derived and extended by W. J. Corlett and D. C. Hague in their article, "Complementarity and the Excess Burden of Taxation."[24] In their attempt to set out the conditions under which a change from direct taxation to indirect taxation[25] would bring about an increase in the supply of work effort and a movement towards a Paretian optimum, Corlett and Hague take as a working hypothesis the notion that the ideal system of taxation is one in which the same ad valorem rate of tax is levied on all goods, including leisure.[26] Given the assumption that the supply of work effort can vary, these two men reach the conclusion that the best system of taxation is one in which high rates of tax are levied on goods that are complementary[27] with

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[23]Ibid., p. 584.

[24]W. J. Corlett and D. C. Hague, "Complementarity and the Excess Burden of Taxation," Review of Economic Studies, Vol. 21 (1953-54), pp. 21-30. Similar mathematical treatment of the problem of direct versus indirect taxation is given by J. E. Meade, Trade and Welfare: Mathematical Supplement (London: Oxford University Press, 1955), pp. 24-46.

[25]An indirect tax on all goods is, as a first approximation, assumed equivalent to an income tax by Corlett and Hague, ibid., p. 22. The possibility of varying degrees of progressivity is, thereby, neglected.

[26]Ibid., p. 26.

[27]Complementarity is used by Corlett and Hague, ibid.,

leisure and low rates of tax on those goods which are competitive with leisure. For in taxing those goods which are most complementary with leisure one is in a sense taxing leisure itself and is, thereby, approaching the ideal system of taxation wherein all goods are taxed.

In the above discussion of the relative merits of direct taxation as opposed to indirect taxation the central problem has been that of achieving a "second best" optimum. The existence of any system[28] of taxation necessarily prevents the satisfaction of one of the Paretian optimum conditions and, accordingly, a Paretian optimum is unattainable. Confronted with the additional constraint of a taxation system, a "second best" solution is only attainable, according to Corlett and Hague, when revenue is raised by a system of unequal indirect taxes with the highest rates of tax being levied on those goods which are most complementary with leisure.

In the field of international trade there has also been witnessed the attempt to derive a position of maximum welfare given the existence of a constraint in addition to those usually assumed operative. In this case, it is the constraint of a tariff which prevents the attainment of a Paretian

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p. 21, to delineate the degree of competitiveness with leisure by which any one good may be characterized.

[28] A poll tax would have no substitution effect; that is, it would only have an income effect.

optimum. In his work with customs unions, Jacob Viner[29] has shown that the formation of a customs union may, in fact, divert production from a low-cost producing, non-member, country to a high-cost, producing, member country. Such diversion of production is referred to by Viner as the "trade-diverting"[30] effect of the customs union. When the "trade-diverting" effect is predominant, as opposed to the trade-creating effect, the formation of a customs union may lead to the injury of one or both of the member countries, and of the whole world.[31] Furthermore, the questions about whether or not the formation of a customs union will yield predominant trade-creating or trade-diverting effects can not "be answered a priori, and the correct answers will depend on just how the customs union operates in practice."[32]

An extension of Viner's work is given by S. A. Osga in his article, "An Essay in the Theory of Tariffs."[33] Osga generalizes Viner's conclusions, with regard to the preferential reduction of tariffs, to the case of non-preferential tariff reductions; that is, the non-preferential reduction

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[29]Jacob Viner, The Customs Union Issue (New York: Carnegie Endowment for International Peace, 1950).

[30]*Ibid.*, pp. 43-4.

[31]*Ibid.*, p. 44.

[32]*Ibid.*, p. 45.

[33]S. A. Osga, "An Essay in the Theory of Tariffs," Journal of Political Economy, Vol. 63 (December, 1955), pp. 489-99.

of tariffs may lead away from the free trade position. In his own words, Osga concludes that "in a world consisting of several countries, each with its own system of tariffs, the removal of some tariffs, no matter whether they are preferential or not, may lead either toward or away from the optimum allocation of the world's productive resources."[34] It should be noted that writer Osga did not believe it was possible to say on a priori grounds whether or not the reduction of tariffs and, in particular, the establishment of a free trade area would lead towards or away from a position of maximum welfare;[35] this is a typical second best conclusion.

Of particular significance in the field of international trade, however, is the work of J. E. Meade[36] and his attempts to derive a second best solution. Meade begins by postulating that not all the conditions for a Paretian optimum are simultaneously fulfilled. He postulates a large number of divergences in the various marginal equalities throughout the economy resulting from taxes, monopoly and monopsony elements and external economies and diseconomies. After further postulating that there is a particular divergence in some marginal condition that it would be possible to eliminate

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[34] _Ibid._, p. 499.

[35] _Ibid._

[36] J. E. Meade, _The Theory of the Customs Union_ (Amsterdam: North Holland Publishing Co., 1955); _Trade and Welfare_ (London: Oxford University Press, 1955); and _Trade and Welfare_,

through government action, Meade poses the question, "Will the reduction of one particular divergence between marginal values and costs in one part of the economy invariably lead to an increase in economic welfare, even though many other divergences between marginal values and costs continue unchanged in other parts of the economy?"[37] His conclusion is that "if there are a number of existing divergences between marginal values and costs, then the reduction of one of these divergences--the others all remaining unchanged--will not necessarily lead to an increase in economic welfare but may very well reduce it."[38]

Meade illustrates his conclusion with an example of the railway and road haulage industries. Both of these industries are assumed to be characterized by a divergence between the marginal value of their service and its marginal cost as a result of, say, taxation or monopoly elements; but the divergence is assumed to be greater in the railway industry. Accordingly, the marginal cost of a given value of transport service is in the railway industry lower than the marginal cost in the road haulage industry.

Now the anti-monopoly policy of reducing the rate of divergence in the road haulage industry might "help to

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Mathematical Supplement (London: Oxford University Press, 1955).

[37] Meade, Trade and Welfare, p. 102.

[38] Ibid.

improve production as between transport services on the one hand and other products on the other hand, if the rate of divergence between marginal values and costs were lower throughout the rest of the industry; for it would encourage people to make greater use of road transport services--which had a high value in relation to their cost--at the expense of other products--which had a lower value in relation to their cost."[39] The effect, however, of such a policy within the transport industry itself might not be so good. For any policy designed to reduce the divergence between the marginal values and costs in the road haulage industry would result in the diversion of transport business from railways to trucking. But the road haulage industry has a higher marginal cost than does the railway industry. Accordingly, if there is active competition between these two industries, the government policy designed to reduce the divergence between the marginal values and costs in the road haulage industry would have the force of directing transport business from the socially inexpensive railway industry to the socially expensive road haulage industry.

In the community as a whole, the government anti-monopoly policy might well reduce collective welfare if the expansion effects are found to be outweighed by the diversion effects. That is, if there was little competition between the transport industry and the remaining industries making up the economy,

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[39]Ibid.

there would be only a small increase in the community's production as a result of the transfer of production to low price transport services from the other products produced in the community, especially if the rate of divergence between marginal values and costs in the industries producing these products was not great. And if there was an original high rate of divergence in the transport industry and especially a high rate in the railway industry, then the government policy would have a great effect in diverting the production of transport services to the road haulage industry with the result that the welfare of the community might very well decline.

With rephrasing, this example of the road haulage and railway industries can be used to examine the possible effects of the removal of a particular tariff given the existence of other tariffs throughout the world which can not be changed. Similar conclusions such as those of Viner and Osga, summarized above, are then derivable. Meade also attempts to answer the question of whether or not welfare might be increased by the purposeful causing of one divergence in order to offset another domestic divergence. Specifically he sets out the problem as follows:

> We now turn . . . to the case in which there is some initial and fixed divergence between marginal values and costs within the domestic economy of one of our trading countries. We assume that there are no other domestic divergences within any of the trading countries and that there are initially no import or export duties and subsidies or any other cause of

> divergence between the marginal values and costs in international trade. Our purpose is to inquire whether in these circumstances the damage to economic welfare caused by the domestic divergence between marginal values and costs might not be in part offset by a departure from free trade, i.e. by introducing a counterbalancing divergence through some commercial policy control over a relevant part of the country's international trade.[40]

Meade's conclusion is that there are some situations in which one particular divergence which can not itself be eliminated may in fact be somewhat offset by the deliberate causing of a divergence between some other set of marginal values and costs.[41]

The problem of tariffs and the effect of the creation of a customs union are also analysed by Lipsey and Lancaster to illustrate the results of a second best type analysis.[42] Using a model similar to the one used in the Little-Corlett and Hague analysis of direct versus indirect taxes, the problem of determining the optimum level of tariffs is tackled first. In this three-commodity model, composed of one domestically produced commodity, Z, and two imported commodities, X and Y, the domestic product of country A is left untaxed. It is assumed, first, that some fixed rate of tariff is levied on the imports of Y. The question is then asked: "What tariff greater than, equal to or less

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[40] Ibid., p. 226.

[41] Ibid., pp. 227-43.

[42] Lipsey and Lancaster, loc. cit., pp. 18-21.

than zero on the imports of commodity X will maximize welfare in country A?"[43] The conclusion is that the welfare of country A will be raised, left unchanged, or lowered by the introduction of a marginal tariff on the originally duty-free imports of X according to whether imports of the commodity Y increase, remain constant or decrease. For these three possibilities, the optimum tariff on the commodity X will be, respectively, positive, zero and negative.

If, instead of assuming that the imports of X were originally imported into country A duty free, it is assumed that the same tariff is levied on the imports of X and Y at a uniform rate, then, the answer to the question of what will be the optimum tariff on X will be concerned with the effect on the consumption of the domestically produced commodity Z. The welfare of country A, in this case, will be raised, left unchanged, or lowered according to whether a marginal increase in the tariff on the commodity X causes a decrease in the consumption of the domestically produced commodity Z, leaves the consumption of Z unchanged, or results in the increased consumption of Z. For these three possibilities, the optimum tariff on the commodity X will be one that is, respectively, greater than the tariff on Y, equal to the tariff on Y, or less than the tariff on Y.

A slight variation of this problem of optimum tariffs is given when it is assumed that a uniform ad valorem rate

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[43] Ibid., p. 19.

of tariff is levied by Country A, the producer of Z, on the imports of commodities X and Y and when it is then assumed that a customs union is formed between Country A and Country B, the producer of X which now enters Country A duty-free. If it is further postulated that the pre-union ad valorem rate of tariff is still levied on commodity Y, the welfare effects on Country A resulting from the formation of the customs union may now be briefly summarized.

Case I: if a fall in the consumption of Y is the result of a small increase in the tariff on X, then the optimum tariff on X is a subsidy and, accordingly, the customs union effectively increases the welfare of Country A.

Case II: if the consumption of commodity Y is left unchanged by changes in the tariff on X, then the optimum tariff on X is equal to zero and, accordingly, the welfare of Country A is effectively increased by the formation of the customs union. If the tariff on Y cannot be varied, then, the optimum tariff of zero on the commodity X constitutes the attainment of a second best optimum.

Case III: if the consumption of commodity Z is left unchanged by changes in the tariff on X, then the equality of the tariffs on X and Y is the condition for second-best optimum welfare and, accordingly, the welfare of Country A is diminished by the formation of the customs union.

Case IV: if the consumption of commodity Z falls as a result of an increase in the tariff on X, then, the optimum

tariff on X is some rate greater than the one levied on Y and accordingly, the welfare of Country A is lowered by the formation of the customs union.

Case V: if the consumption of both Y and Z increases as a result of an increase in the tariff on X, then, the optimum rate of tariff on the commodity X is less than that on Y but is greater than zero and, accordingly, the effect on the welfare of Country A, resulting from the formation of the customs union is unknown.

The typical conclusions of second-best type analysis are in evidence. Very little can be said a priori about the removal of a tariff and its effect on the community welfare given the fact that all tariffs cannot be removed.

There is, however, one tentative conclusion that is reached concerning the welfare effects resulting from the various stages of the removal of a tariff. If the tariff is removed in stages, the welfare of Country A will generally be raised by the initial reductions and lowered by the latter stages of reduction. This conclusion of Lipsey and Lancaster is analogous to that reached by Meade in his analysis; indeed, the Lipsey-Lancaster conclusion is somewhat clarified by Meade's statement as follows:

> there is reason to believe that in the formation of a customs union--at least in the simple case now under examination--the first stages of preferential reduction of duties on the mutual trade of the partners is more likely to be beneficial than the last stages of elimination of their mutual duties. The mutual trade which is stimulated by the first preferential tariff reductions will be subject to a higher welfare

> weight than the mutual trade which is stimulated by the final elimination of their mutual duties. Unless, therefore, there is some special reason to believe that more contraction of trade will be associated with the first stages of the formation of the union than with the latter stages, it follows that the first stages are likely to do more good (or do less harm) than the final stages of the creation of the union.[44]

However, the total effect on welfare resulting from the complete removal of the tariff on X, given that a marginal increase in the tariff would increase the consumption of Z and Y, cannot be specified.

The conclusions of Meade, Viner, Osga and Lipsey and Lancaster about the problems of finding a "second-best" solution in the field of international trade are, thus, similar to those reached by MacKenzie in dealing with the problem of the optimum levels of output in multi-input industries with inter-firm sales, to those reached by Smithies in the problem of maximizing production and utility, and, also, to those reached by Little and Corlett and Hague in their analysis of the best way to raise revenue by the alternative methods of indirect and direct taxation. That is, these several problems of maximization subject to the operation of one or more additional constraints yield a general method of approach to any such problem of determining a doubly constrained maximum. There are, however, certain important distinctions which can be made

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[44] Meade, The Theory of Customs Unions, pp. 50-1.

between these various maximization problems.

It has already been noted that certain of these examples dealt with the difficulties involved in the determination of a Paretian optimum while others dealt with the difficulties involved in the determination of a second-best solution, made necessary because of the preclusion of a Paretian optimum. Specifically, the problems of profit and utility maximization, dealt with by Smithies, were problems associated with the determination of a Paretian optimum. In both of these examples simple production and utility functions were used, respectively, to define a Pareto-optimal production and exchange position. The postulated operation of certain additional constraints, which should have been included in the original incomplete production and utility functions, resulted in the determination of a new Pareto-optimal production and exchange position. These new production and exchange optima were then shown to be defined by a new and more complex set of marginal conditions.

In contrast, the problems dealt with by MacKenzie, by Meade, Viner, Osga, and Lipsey and Lancaster, and by Little and Corlett and Hague were all problems associated with the determination of a second-best optimum. For in these problems the operation of additional constraints prevented the attainment of a Paretian optimum.

Specifically, the existence of a tax prevents the satisfaction of the optimum condition that states that subjective rates of substitution must be equal to technical rates of transformation, and, accordingly, a Pareto-optimal allocation of productive resources as among possible uses is prevented; the existence of a divergence between price and marginal cost in any one firm or industry precludes the attainment of Pareto-optimal levels of production within the economy; and, finally, the existence of a tariff prevents the attainment of a Pareto-optimal allocation of production as among various countries.

A further distinction can be made among these various examples according to the level of welfare or utility that is to be maximized; that is, among those problems dealing with the determination of a welfare maximum by an individual firm or consumer, those by a single country or economy, and those by the entire world community.

The problems of finding either a production or exchange optimum by the individual firm or consumer pose no great difficulty to the policy maker. For in both cases, the assumption of profit maximization on the part of the individual firm and the assumption of utility maximization on the part of every consumer imply that the firm or consumer will automatically compensate for any additional constraints. If it is in the power of the firm or consumer to satisfy the set of conditions defining,

what for him is a production or exchange optimum, respectively, then the individual firm or consumer is assumed to satisfy these conditions automatically and to attain, thereby, what for him may be called a Pareto-optimal position of production or exchange. On the other hand, if it is not in the power of the individual firm or consumer to satisfy all the conditions defining his production or exchange optimum, respectively, then, the individual firm or consumer will not be able to attain a Pareto-optimal position. Rather, a second-best position will be sought and the individual firm or consumer will probably find that this second-best position will only be attained if certain marginal values and costs, other than those outside his power to equate, are allowed to diverge to varying degrees. In any case, the firm or individual will automatically arrive at a maximum position, be it a Paretian one or a second-best one, given the assumptions of profit and utility maximization.

Now if the community, either the national or the international community, could be assumed to automatically maximize its welfare, then, problems of welfare maximization at this level would pose no problems as far as the policy maker is concerned. But, unlike the individual firm or consumer, the community cannot be conceived of as automatically maximizing its welfare; as Mishan states, "on the community level, however, where it is no longer

legitimate to suppose a unique utility function and to ignore distribution of the product, there can be no assumption of maximization."[45] It is into this category of maximization problems that fall those of determining the optimum pricing policy for the various industries of an economy given the condition that there is one industry in which there exists a divergence between price and marginal cost which cannot be removed; those of determining the optimum system of taxation given the necessity of raising revenue by some method of taxation; and, at the international level, those of determining the optimum system of tariffs given the existence of one tariff which cannot be removed.

In each of these problems the attainment of a second-best optimum has been shown to require the disruption of optimum levels of production and exchange reached at a lower level. That is, in order to compensate for a divergence between marginal cost and price in one industry and to reach a second-best optimum, it will be necessary to disturb optimum levels of production in other industries, characterized by price equal to marginal cost, by deliberately violating this optimum condition. Similarly, the optimum system of tariffs given the existence of one tariff is a system in which there are other tariffs of varying degrees. And even though a particular country has arrived at a maximum position

[45]E. J. Mishan, "Second Thoughts on Second Best," Oxford Economic Papers, Vol. 14 (October, 1962), p. 209.

relative to itself, in order to maximize world-welfare, it may be necessary for this particular country to introduce certain tariffs.

The significance of these maximization problems at the community level is obvious. It is in these areas that national, and, indeed, international, policy is required. But in order to set out national policy, there is need of criteria with which to judge the appropriateness of any particular policy.

The examples of maximization problems discussed in this chapter yield certain general conclusions which may be summarized briefly. They are two-fold: first, if a Paretian optimum is incapable of achievement because one of the conditions satisfying this optimum can itself not be satisfied, then, a second-best optimum will likely be achieved by deliberately not satisfying certain of the other optimum conditions; and, second, the conditions defining this second-best optimum are likely to be quite complex. The appropriateness of the Paretian optimum conditions as guides to policy is thus seriously questioned.

It is from these general conclusions and, in particular, from the observed similar method of approach to the determination of a doubly-constrained maximum, that a general theory of second-best has been developed. Accordingly, to the work of Lipsey and Lancaster in their

article, "The General Theory of Second Best,"[46] attention is turned in the following chapter.

[46] Lipsey and Lancaster, loc. cit., pp. 11-32.

CHAPTER IV

THE GENERAL THEORY OF SECOND BEST

The General Theorem of Second Best[1] as set out by Lipsey and Lancaster in their article, "The General Theory of Second Best,"[2] states that "if there is introduced into a general equilibrium system a constraint which prevents the attainment of one of the Paretian conditions, the other Paretian conditions, although still attainable, are, in general no longer desirable."[3] Following directly from this theorem are two important negative corollaries. The first negative corollary states that one situation, in which all the Paretian marginal conditions are not fulfilled but in which more of these conditions are fulfilled than are in a second situation, is not necessarily better, in the sense of being a closer approximation to the Paretian optimum, than the second situation;

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[1]Lipsey and Lancaster have taken the name, "Theory of Second Best," from the writings of J. E. Meade in Trade and Welfare (London: Oxford University Press, 1955). Meade, ibid., p. 8, defines a second-best criterion as opposed to a utopian criterion, as one which assumes all other policies to remain unchanged, even though these policies are not of an optimum nature. The particular act of policy, then, is directed towards one particular divergence between the marginal conditions.

[2]R. G. Lipsey and K. Lancaster, "The General Theory of Second Best," Review of Economic Studies, Vol. 24 (1957), pp. 11-32.

[3]Ibid., p. 11.

that is, "there is no a priori way to judge as between various situations in which some of the Paretian optimum conditions are fulfilled while others are not."[4] Similarly, the second negative corollary states that one situation, in which the Paretian conditions are not all satisfied but in which the divergences between the marginal conditions are equal in direction and magnitude, is not necessarily superior to a similar situation in which the direction and magnitude of the divergences are not equal; that is, "there is no a priori way to judge as between various situations in which none of the Paretian optimum conditions are fulfilled."[5] Taken in combination, then, the general theory of second best and its two negative corollaries would appear to represent a direct, and, indeed, overwhelming, attack on the non-discriminatory application of piece-meal welfare economics.

The formal development of the general theory of second best may now be outlined briefly.[6] Define a function of n variables as $X_1, \ldots, X_n$:

(1) $F(X_1, \ldots, X_n)$.

This function is to be maximized subject to the constraint imposes on the n variables by the following function defined

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[4] Ibid., p. 12.

[5] Ibid., p. 12.

[6] This development and following proof follow closely the work of Lipsey and Lancaster, ibid., pp. 26-7.

as:

(2) $\emptyset(X_1, \ldots, X_n) = 0$

Maximization of equation (1) subject to the constraint imposed by equation (2) will yield the marginal first order conditions for a Paretian optimum. The $(n - 1)^{th}$ condition, defined as

(3) $G^i(X_1, \ldots, X_n) = 0 \quad (i = 1, \ldots, n - 1),$

may be called the solution to this maximization problem. The general theory of second best is now capable of mathematically rigorous definition as follows:

> If there is an additional constraint imposed of the type $G^i \neq 0$ for $i = j$, then the maximum (minimum) of F subject to both the constraint $\emptyset$ and the constraint $G^i \neq 0$ will, in general, be such that none of the still attainable Paretian conditions $G^i = 0$, $i \neq j$, will be satisfied.[7]

A brief summary of the proof of this theorem will follow directly.

If the additional constraint is inoperative, the Paretian conditions may be found by use of the Lagrangean multiplier. The maximization problem may, accordingly, be set out as:

(4) $F_i - u\,\emptyset_i = 0 \qquad (i = 1, \ldots, n)$

from which may be derived the following set of n equations:

(5) $\frac{\partial F}{\partial X_1} = u.\frac{\partial \emptyset}{\partial X_1}\,;\ \frac{\partial F}{\partial X_2} = u.\frac{\partial \emptyset}{\partial X_2}\,;\ \ldots\,;\ \frac{\partial F}{\partial X_n} = u.\frac{\partial \emptyset}{\partial X_n}\,.$

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[7] _Ibid._, p. 26.

If the Lagrangean multiplier u is eliminated this set of n equations represented by equation (5) may be reduced to a set of n - 1 equations:

$$u = \frac{\partial F}{\partial X_1} \cdot \frac{\partial X_1}{\partial \emptyset} = \frac{\partial F}{\partial X_2} \cdot \frac{\partial X_2}{\partial \emptyset} = \ . \ . \ . \ = \frac{\partial F}{\partial X_n} \cdot \frac{\partial X_n}{\partial \emptyset} \ .$$

i.e.

$$(6) \quad \frac{\frac{\partial F}{\partial X_1}}{\frac{\partial \emptyset}{\partial X_1}} = \frac{\frac{\partial F}{\partial X_2}}{\frac{\partial \emptyset}{\partial X_2}} = \ . \ . \ . \ = \frac{\frac{\partial F}{\partial X_n}}{\frac{\partial \emptyset}{\partial X_n}} \ .$$

Defining the first term of the above set of equations as $\frac{F_1}{\emptyset_1}$ and similarly the second and following terms as $\frac{F_2}{\emptyset_2}$, . . ., $\frac{F_n}{\emptyset_n}$, this set of equations (6) may be set out in the following manner:

$$\frac{F_1}{\emptyset_1} = \frac{F_2}{\emptyset_2} = \ . \ . \ . \ = \frac{F_n}{\emptyset_n}$$

or more concisely as:

$$(7) \quad \frac{F_i}{\emptyset_i} = \frac{F_n}{\emptyset_n} \qquad (i = 1, \ . \ . \ ., \ n-1)$$

This set of (n-1) equations expressed by equation (7) may by simple transposition be made to take the following form:

$$(8) \quad \frac{F_i}{F_n} = \frac{\emptyset_i}{\emptyset_n}$$

or $$G_i = \frac{F_i}{F_n} - \frac{\emptyset_i}{\emptyset_n} = 0 \qquad (i = 1, \ . \ . \ ., \ n-1).$$

In this above form, where F was defined as the utility function of an individual and where $\emptyset$ was defined as his transformation function, the set of equations expressed by equation (8) would indicate the marginal or first-order conditions of a Paretian optimum. Defining X_i and X_n as products, this set of equations would set out the fifth marginal condition which states that the marginal rate of substitution between any two products X_i and X_n for all consumers who purchase both must be the same as the marginal rate of transformation between the same two products for all firms who produce both products. By reinterpreting the symbols X as different factors of production or as factors and products or any one of these combinations over time, all the seven marginal conditions outlined in Chapter 1 may be set out.[8]

If now a constraint in addition to the usual one expressed by equation (2) is assumed operative, the Paretian optimum is made unattainable. This additional constraint may be conceived of as preventing the satisfaction of one of the marginal conditions expressed by equation (8); it may be written as:

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[8]The derivation and interpretation of these sets of equations is set out by J. de V. Graaff, Theoretical Welfare Economics (Cambridge: Cambridge University Press, 1957); James M. Henderson and Richard E. Quandt, Microeconomic Theory (New York: McGraw-Hill Book Co., Inc., 1958), pp. 201-24; O. Lange, "The Foundations of Welfare Economics," Econometrica, Vol. 10 (July, 1942), pp. 215-28; and P. A. Samuelson, Foundations of Economic Analysis (Cambridge: Harvard University Press, 1947), pp. 229-49.

$$\frac{F_1}{F_n} = k.\frac{\emptyset_1}{\emptyset_n} \quad \text{or} \quad \frac{\frac{\partial F}{\partial X_1}}{\frac{\partial F}{\partial X_n}} = k \frac{\frac{\partial \emptyset}{\partial X_1}}{\frac{\partial \emptyset}{\partial X_n}} ;$$

$$(9) \quad \text{or} \quad H = \frac{F_1}{F_n} - k\frac{\emptyset_1}{\emptyset_n} = 0 \qquad k \neq 1$$

where k is, for convenience, assumed constant.[9] With the imposition of this additional constraint the maximization expression now takes the following form:

$$(10) \quad F - u' \emptyset - vH .$$

By substitution, equation (10) becomes:

$$(11) \quad F - u' \emptyset - v(\frac{F_1}{F_n} - k\frac{\emptyset_1}{\emptyset_n})$$

where u' and v are Lagrange multipliers. Differentiating the Lagrange maximizing expression of equation (11) with respect to X, the following first order equations are derived:

$$(12) \quad \frac{F}{X_i} - u'\frac{\emptyset}{X_i} - v \frac{F_nF_{1i} - F_1F_{ni}}{F_n^{\,2}} - k\frac{\emptyset_n\emptyset_{1i} - \emptyset_1\emptyset_{ni}}{\emptyset_n^{\,2}} = 0$$

$$(i - 1, \ldots, n).$$

Define Q_i as the expression $(\frac{F_nF_{1i} - F_1F_{ni}}{F_n^{\,2}})$; and define R_1

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[9]Lipsey and Lancaster, loc. cit.

as the expression $\left(\frac{\phi_n \phi_{1i} - \phi_1 \phi_{ni}}{\phi_n^{\,2}}\right)$. Equation (12) may now be rewritten as:

$$(13) \quad F_i - u'\phi_i - v(Q_i - k\,R_i) = 0$$

By algebraic manipulation, equation (13) may be written as:

$$(14) \quad \frac{F_i}{F_n} = \frac{\phi_i + \frac{v}{u'}(Q_i - kR_i)}{\phi_n + \frac{v}{u'}(Q_n - kR_n)} \;;$$

for $\qquad F_i - v(Q_i - kR_i) = u'\phi_i$

and by transposition: $F_i = u'\phi_i + v(Q_i - kR_i)$.

By dividing by u':

$$(15) \quad \frac{F_i}{u'} = \phi_i + \frac{v}{u'}\,(Q_i - kR_i) \;.$$

Similarly:

$$(16) \quad \frac{F_n}{u'} = \phi_n + \frac{v}{u'}\,(Q_n - kR_n) \;.$$

Taking equations (15) and (16) in combination the following expression, equation (14), is obtained:

$$\frac{F_i}{F_n} = \frac{\phi_i + \frac{v}{u'}(Q_i - kR_i)}{\phi_n + \frac{v}{u'}(Q_n - kR_n)} \;.$$

or alternately:

$$(17) \quad \frac{F_i}{F_n} = \frac{\phi_i}{\phi_n} \quad \frac{1 + \frac{v}{u'} \cdot \frac{(Q_i - kR_i)}{\phi_i}}{1 + \frac{v}{u'} \cdot \frac{(Q_n - kR_n)}{\phi_n}} \;.$$

Accordingly, given the imposition of the additional constraint

as expressed by equation (9), the conditions for a second-best optimum, analogous to the conditions defining a Paretian optimum as set out in equation (8), are expressed by equation (17). These second-best conditions will be equivalent to the conditions necessary for the Paretian optimum if the following expression from the right-hand side of equation (17) is equal to unity:

i.e. if:

$$(18) \quad \frac{1 + \frac{v}{u^T} \cdot \frac{1}{\emptyset_i} (Q_i - kR_i)}{1 + \frac{v}{u^T} \cdot \frac{1}{\emptyset_n} (Q_n - kR_n)} = 1 .$$

If v = 0, equation (18) will be satisfied; and, similarly, if the quantity $(Q_i - kR_i)$ is equal to the quantity $(Q_n - kR_n)$ it will also be satisfied. But v cannot equal zero; for, if it did, the expression $\frac{F_1}{F_n}$ would equal the expression $\frac{\emptyset_1}{\emptyset_n}$ where i = 1. Such a situation would contradict the original hypothesis of the existence of the additional constraint which took the form of these two expressions not being equal to zero.

Examination of the following expressions:

$$Q_i = \frac{F_n F_{1i} - F_1 F_{ni}}{F_n^2}$$

$$Q_n = \frac{F_n F_{1n} - F_1 F_{nn}}{F_n^2}$$

$$R_i = \frac{\emptyset_n \emptyset_{1i} - \emptyset_1 \emptyset_{ni}}{\emptyset_n^2}$$

$$R_n = \frac{\phi_n \, \phi_{1n} - \phi_1 \, \phi_{nn}}{\phi_n^{\,2}}$$

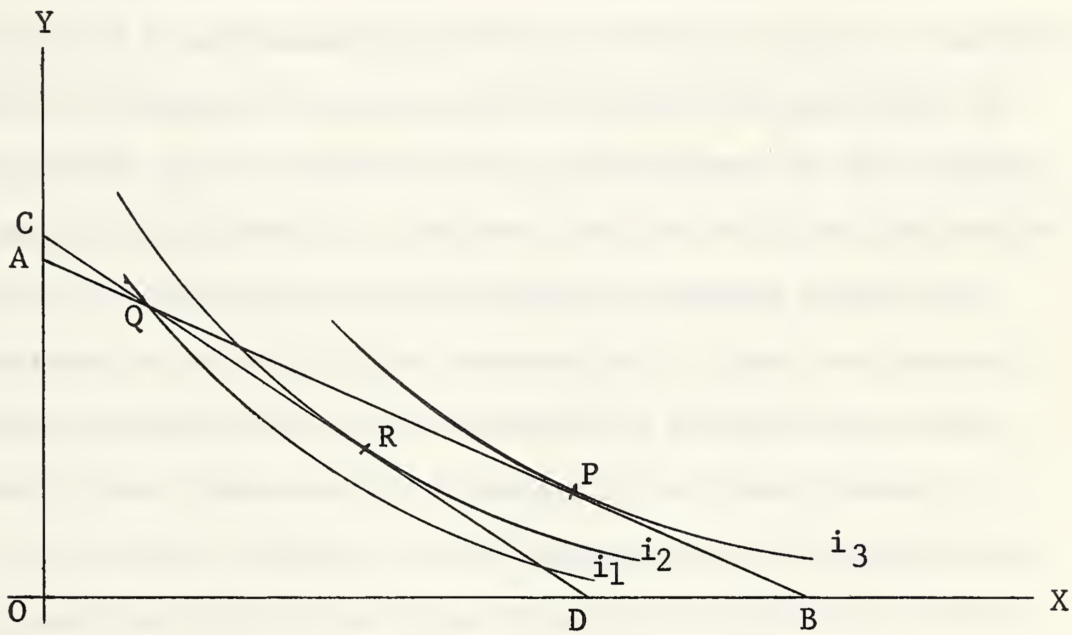
lead Lipsey and Lancaster to the conclusion "that nothing is known, in general, about their signs, let alone their magnitudes, and even the signs would not be sufficient to determine whether (ii) was satisfied or not,"[10] where case (ii) is the condition that $v \neq 0$ but that $(Q_i - kR_i)$ equal $(Q_n - kR_n)$. The second-best optimum is then defined by a set of conditions of the type expressed by equation (9) and, generally, the usual Paretian marginal conditions will be left unsatisfied and, indeed, "will be broken all around."

Using the diagram of Figure IV, an illustration of the second-best solution can be seen. The X axis measures quantities of the good X and the y axis quantities of good y. The line AB is the transformation function. Without any additional constraint besides that imposed by the usual transformation function, the Paretian optimum would be achieved at the tangency solution of the community's welfare contour line i_3 and the transformation curve, AB, at the point P. With the imposition of an additional constraint represented by the line CD, the Paretian optimum is no longer attainable; for only points lying on or below CD are capable of achievement. The point Q on the original transformation function AB is still attainable but is no longer

[10]<u>Ibid.</u>, p. 27.

desirable. Rather, the point R lying below the original transformation function is the second-best solution. It might be noted here that a thorough-going geometric presentation of the general theory of second-best has been developed with the aid of the concepts of production-possibility-indifference curves and budget-possibility-indifference curves[11] by M. McManus in his article, "Comments on the General Theory of Second Best."[12]

Figure IV.--Diagramatic Illustration of Second-Best Analysis.



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[11]These production-possibility-indifference curves and budget-possibility-indifference curves are developed by M. McManus in "The Geometry of Point Rationing," Review of Economic Studies, Vol. 22 (1954-55), pp. 1-14; and are used by McManus to analyze the problem of rationing which is itself a second-best phenomenon.

[12]M. McManus, "Comments on the General Theory of Second-Best," Review of Economic Studies, Vol. 26 (1959), pp. 209-24.

It is expedient at this stage to emphasize the broad scope of the general theory of second best. For it is truly a general theory setting out, as it does, the general method of approach to, and the general conclusions which result from, the determination of a second-best type maximal solution given the preclusion of a first-best maximal solution because of the impossibility of satisfying one or more of the conditions which define a first-best solution. Profit maximization subject to the operation of a constraint in addition to the usual cost constraint is, thus, an example of a maximization problem characterized by the same method of approach and one which yields the same type of conclusions as the approach and conclusions of the general theory of second-best. The same may be said for the maximization of utility by the individual consumer given the existence of an additional constraint. These two maximization problems have been examined by Smithies and have already been discussed in Chapter III of this thesis.

It is not, however, with these types of maximization problems that this thesis is primarily concerned. For in fact, given the assumptions of profit and utility maximization on the part of the individual firm and consumer, respectively, the individual firm and consumer will automatically attempt to reach a maximum position by satisfying whatever conditions that it is necessary to satisfy in order that this maximum position be attained. As was stated in

Chapter III, if the individual firm or consumer is able to satisfy all the various conditions defining a maximum solution, he will so satisfy these conditions and will arrive at what might be termed a Pareto-optimal position of production and exchange, respectively. Accordingly, the individual firm and consumer of the two Smithies examples would reach a Pareto-optimal position of production and exchange even though it was necessary for the individual firm or consumer to satisfy a more complex set of conditions defining the additionally constrained maximum.

There are also maximization problems at the community level to which the conclusions of the general theory of second-best are comparable and applicable. In the problem of setting out an optimum taxation scheme, a Pareto-optimal position is precluded because of the varying existence of any taxation scheme, other than a poll tax. In order that a Pareto-optimal position be attained it has been shown in Chapter III, that all taxes direct and indirect would have to be repealed. For only then would the citizens of any particular community be able to satisfy all the marginal equivalences defining the optimum allocation of production within the community. Now even though the various economic units, be they individual consumers or business firms, will attempt to reach a maximum position for themselves by compensating for taxes and, indeed, will satisfy the new conditions defining their, respective, constrained production

and exchange optima, the community as a whole faces the problem of the optimum system of taxes to levy in order that a second-best optimum allocation of production within the community be determined. The optimum system of taxes has been shown in Chapter III to be one in which direct and indirect taxes of varying rates are levied throughout the community. This conclusion is thus a typical second best conclusion.

Perhaps the most important maximizing problems which lend themselves to treatment by a second-best type analysis are those concerned with the optimum pricing system which would ensure the optimum allocation of resources within any one country and those concerned with the optimum system of tariffs which would ensure the optimum allocation of resources within the world community. For, as in the case of the optimum system of taxation, there is the necessity of government policy to ensure that a maximum position be attained in the individual community and in the world at large. These problems of maximization at the community level arise because, as has been noted in Chapter III, the community itself cannot be assumed to automatically maximize its own welfare.

At the national level in any one economy there is the problem of achieving an optimum allocation of resources in the face of institutional and monopoly-power constraints. In Chapter II, the optimum allocation of resources was found

to require the necessary equality of price to corresponding marginal cost for every product and for every factor. A Paretian optimum allocation of resources was thus achieved when the price was everywhere equal to the corresponding marginal cost. If, however, there is one firm or one industry in which price does not equal marginal cost and because of institutional rigidities an equality can not be obtained between these diverging values, then a Paretian optimum is unattainable and it is a second best allocation of resources that is sought. The conclusions of the general theory of second best can thus be applied appropriately to this maximization problem. In all probability the attainment of a second best optimum will require that certain optimum levels of production reached by various individual firms will have to be distorted in order that a second-best optimum for the economy as a whole be found.

Similarly, on the international level and, specifically, as between the optimum allocation of production as between the various countries of the world, it is not possible, as it was in the case of the individual firm and consumer, to assume that the world community will automatically maximize its welfare. Now a Pareto-optimal allocation of production between the various countries would be attained if the national government policy of each and every nation of the world was such as to result in the absence of all trade restrictions be they quotas or tariffs. If, however, there

is one or more trade restrictions, say one or more tariffs, which, because of some institutional rigidities, it is impossible to remove, and, thereby, if a Paretian optimum is unattainable, then, it is necessarily a second best solution that is sought. The conclusions of the general theory of second best can thus be also applied appropriately to this world-welfare maximization problem. Accordingly, in all probability the second best solution will require that certain countries, which hitherto might have attained an optimum allocation of production in their own country without the necessity of levying any tariff, must now introduce various tariffs to compensate for the one tariff which cannot be removed. The optimum allocation of production initially observed in each of the various countries may now have to be distorted in order that the world community as a whole compensates for the additional constraint of one or more irremovable tariffs. These conclusions are, thus, similar to those reached by Meade, Viner, Osga and Lipsey and Lancaster as noted and discussed in Chapter III

The conclusions of the general theory of second best are thus seen to be relevant to several types of maximization problems in which the operation of a constraint in addition to those usually assumed operative precludes the attainment of a first-best type solution and necessitates the determination of a second best solution. It is, however, with the welfare-maximizing problems of the community level,

be that national or international, that this thesis is primarily concerned. For it is in these maximization problems that the assumption of automatic welfare maximization cannot be made and, accordingly, in which the conclusions of the general theory of second best would appear to have their most important consequences.

Further clarification of the theory of second best and an indication of its practical applicability may be obtained from an examination of a situation which gives rise to the possibility of the achievement of a second-best optimum. The problem to which Lipsey and Lancaster apply their second-best analysis is that of determining the correct pricing policy which should be followed by a nationalized industry in an economy in which there exists a monopoly.[13] The question posed for answering is: "What is the best pricing policy for the nationalized industry to pursue in order to maximize the welfare of the community if it is assumed that the mixed economy is comprised of a nationalized industry and a monopoly which cannot be removed?" In a full-employment situation, the monopoly will be producing at a level of output below that considered to be an optimum in the Paretian sense; while the non-monopoly firms will be producing at a level beyond the optimum. With the nationalization of one of the non-monopolized industries, there are two possible lines of pricing policy open to the nationalized

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[13]Lipsey and Lancaster, <u>loc. cit.</u>, pp. 21-5.

industry: it may continue to operate in a competitive manner or it may behave more monopolistically, producing less than is considered optimal and, thereby, indirectly contributing to levels of production that are in excess of the optimum level. Either policy will not bring about a Paretian optimum; it is a second-best optimum that is sought.

A three-good, three-industry model is built wherein the good X is produced by a monopoly, Z is produced competitively and Y, originally produced competitively, is now produced by the nationalized industry. The monopolist's behavior is characterized by his production of X at a level which sees the price of X diverging from the marginal cost of X. On the other hand, the price of Z is equal to the marginal cost of Z. In order to find the optimum price of Y which will yield a second-best optimum it is necessary to maximize the community's preference function subject to the two constraints of the original transformation function and the postulated behavior of the monopolist. The community's preference function is assumed to define the "public interest" and is a function of the three goods X, Y and Z. When this utility function is maximized subject to the two constraints, the conditions for attaining a second-best optimum are found to require that "the nationalized industry should set its price higher than its marginal cost and, to that extent, behave like a monopoly."[14]

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[14] Ibid., p. 25.

Further consideration of the pricing policy of the nationalized industry as compared to the monopoly yields the conclusion that the nationalized industry should follow a pricing policy that is intermediate between that followed by the competitive and monopoly industries. Accordingly, the correct pricing policy for the nationalized industry required that, "on the one hand, it should set its price higher than marginal cost (relative to the numéraire) but, on the other hand, it should not set its price so far above marginal cost as is the case in the monopolized industry."[15]

It might be mentioned here that the mathematical and methodological soundness of Lipsey and Lancaster's model of a nationalized industry in a mixed economy and, specifically of their postulated maximization subject to only one additional constraint has been questioned. Both Mishan[16] and McManus[17] have argued that the presence of only one additional constraint, say that of the monopolist setting the price of his product above the marginal cost of the product according to some fixed ratio, leads to no problem in the attainment of a Paretian optimum. For the correct pricing problem is that which would make the ratios of price

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[15] Ibid.

[16] E. J. Mishan, "Second Thoughts on Second Best," Oxford Economic Papers, Vol. 14 (October, 1962), pp. 207-8.

[17] McManus, "Comments on the General Theory of Second Best," pp. 210-3.

to marginal cost in the remaining sectors equivalent to the price-marginal cost ratio set by the monopolist. Given this three-good, three industry model of Lipsey and Lancaster with the postulated one input and constant costs, the necessary condition for the attainment of a Paretian optimum, is then, the application of the Proportionality Rule.

It is further argued that in order to be faced with a situation in which a true second-best optimum must be sought, it is necessary to postulate the existence of at least two constraints in addition to the usual transformation function. Accordingly, in the problem under consideration, in order for the correct pricing policy of the nationalized industry to be of a second-best type it would be necessary to postulate that not only is it impossible to eliminate the divergence between the price and marginal cost of the products produced by the monopoly but also that it is impossible to change the pricing policy of the competitive industry. That is, it must be assumed that the price of the competitively-produced products cannot be made to diverge from the marginal cost of these products.

These objections of Mishan and McManus appear, however, to have been satisfactorily rebutted by Lipsey and Lancaster.[18]

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[18] Lipsey and Lancaster, "McManus on Second Best," Review of Economic Studies, Vol. 26 (1959), pp. 225-6. The excessive concern of McManus with the mathematical definition of the additional constraint appears to have allowed him to pass over the significance of the constraint as a description of monopoly-pricing policy.

Given the government action of setting the prices in the industries of the economy other than the monopolized industry above marginal costs to the same extent that the monopoly has set its price above marginal cost, the monopolist will find that, as the price of his product falls relative to the prices of the other products produced in the economy, the quantity of his product demanded will increase. This increase in the quantity demanded will tend to move the monopolist away from that level of output at which he maximizes profit and, accordingly, as long as his increased production is not characterized by rapidly decreasing costs, he will attempt to maintain a position of maximum profit by raising his price.

This raising of the price of the monopoly-produced good, however, has the effect of increasing the rate of divergence between price of this good and its marginal cost. Herein lies the problem which gave rise to the objections of Mishan and McManus. In their three-industry model, Lipsey and Lancaster, as a first approximation, had interpreted the postulated additional constraint of the monopolist's pricing policy as one which required the price-marginal cost ratio of the monopoly-produced good being fixed; that is, "the monopoly will set the price of X higher (in terms of the numéraire, which will be taken to be Z) in relation to marginal cost than in the conditions of the Paretian optimum."[19] In order to overcome the objections,

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[19] Lipsey and Lancaster, "The General Theory of Second Best," p. 23.

it is necessary to reinterpret the behavior of the monopolist. Rather than gear his pricing policy to a fixed ratio of price to marginal cost or to the pricing policy of any one particular industry, such as would be the case if he set his price at some constant k times the price of the goods produced in the competitive sector, the monopolist's pricing should be interpreted as being related to some general overall average of the price-marginal cost ratios of all the other industries with "the functional relationship being ultimately determined by demand conditions."[20] In sum, it may be stated that the single additional constraint of the existence of monopoly-type pricing policy in one industry is sufficient to preclude the attainment of a Paretian optimum and to require that a second-best optimum be sought.

In the two examples, presented by Lipsey and Lancaster, of second-best theory applied to the determination of practical policy, simplifying assumptions made it possible to derive certain general conclusions. In the three-good, three country model discussed in Chapter III, the analysis of the second-best optimum system of tariffs was carried on subject to the assumptions that all the individuals of Country A possessed identical homogeneous utility functions which could be summed to find the community welfare function; that the taxes and tariffs of Country A did not affect the prices

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[20] Lipsey and Lancaster, "McManus on Second Best," p. 226.

of the two imported commodities X and Y in terms of the domestically produced commodity Z; and, that domestic producers of X and Y were not protected by the tariffs of Country A on imports of these two goods X and Y, the first of which is not produced by Country C and the second of which is not produced by Country B, thereby, excluding the possibility of trade-diverting effects.[21] Similarly, in the above-discussed model of the nationalized industry simplifying and specific assumptions were made, such as the fixed total supply of the one input labor, constant costs and a community preference function defining the "public interest" of a community composed of individuals with identical homogeneous utility functions. However, when it is not possible to specify anything but a generalized utility function, the difficulties involved in making a priori statements about policy which would yield a second best optimum are many.

Lipsey and Lancaster attempt to simplify the problem of determining the nature of a second-best optimum by assuming that the function F defines a utility function and that a transformation function is defined by the linear function Ø.[22] The assumption of the linearity of the

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[21] Lipsey and Lancaster, "The General Theory of Second Best," p. 19.

[22] McManus, "Comments on the General Theory of Second Best," p. 220, points out that the well-behaved utility and transformation functions, postulated by Lipsey and Lancaster, effectively exclude the possibility of corner

transformation function effectively eliminates the second derivatives of $\emptyset$ from the conditions defining a second best optimum as expressed by equation (17) and, in particular, by the expressions R_i and R_n. The conditions for a second best optimum now may be written as follows:

$$(19) \quad \frac{F_i}{F_n} = \frac{\emptyset_i + \frac{v}{u'} \cdot \frac{Q_i}{\emptyset_i}}{\emptyset_n + \frac{v}{u'} \cdot \frac{Q_n}{\emptyset_n}}$$

There still remain, however, the second order derivatives of the utility function F as expressed by the expression Q_i and Q_n.

An examination of these second-order derivatives and especially of eight particular cases leads Lipsey and Lancaster to the conclusion that there is only one situation in which an a priori statement can be made about the nature of the second-best optimum. In the context of the model of a nationalized industry in an economy with a monopolized industry and a competitive industry, this sole determinate case yields the conclusion that "if the monopolized commodity is complementary (in the Edgeworth-Pareto sense) to the numéraire, and the i^{th} commodity is also complementary to the numéraire, but a substitute for the

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optima, thereby, narrowing the scope of the general theory of second best to the problems of determining internal second best optimum only. McManus' own geometric treatment of the theory of second best allows for the attainment of second-best corner optima, ibid., p. 219. 1.

monopolized good, then, in order to attain a second-best solution, the price of the i^{th} commodity must be set higher than its marginal cost."[23]

By making the "more heroic" assumption that the two commodities under consideration, the one produced by the monopolist along with the one produced by the nationalized industry, are "weakly related," Lipsey and Lancaster are able to derive a total of three determinate cases from the several cases they examine. These are summarized in the statement that "if the monopolized good and the numéraire are either complements or only weakly related, then the second best solution will certainly require the price of the i^{th} good to be set above its marginal cost either if the good is a substitute for the monopolized good and either complementary or only weakly related to the numéraire, or if the good is weakly related to the monopolized good but complementary to the numéraire."[24] And, finally, no determinate cases are found in a situation in which complementarity between any pairs of goods is absent and in which there exists a strong, or at least not weak, relationship between the monopolized commodity and the numéraire.[25]

Similar conclusions to these above-mentioned ones of

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[23]Lipsey and Lancaster, "The General Theory of Second Best," p. 30.

[24]*Ibid.*

[25]*Ibid.*, pp. 30-1.

Lipsey and Lancaster are reached by H. A. John Green in his analysis[26] of a similar problem. Explicitly introducing a budget constraint into his analysis, Green attempts to discover what the optimal price-marginal cost ratios would be in certain industries given the presence of monopoly and taxation and, thereby, given the condition that the price-marginal cost ratios are fixed in other industries; and also, subject to these same circumstances, what the optimum ratio of labor income after tax to labor income before tax would be. He arrives at the following conclusions:

> (a) If some of these ratios are given and others are free to vary, the first-order conditions for an optimum prescribe that, if commodities are substitutes in the Hicksian sense, the variable ratios should lie within the range bounded by the largest and smallest of the given ratios. If complementarity is present, no such bounds can in general be set for the optimal ratios.
>
> (b) Consider a set of such ratios, some given and some variable, the latter set at values prescribed by the first-order conditions. Let us now change one of the variable ratios (say that of X_i). If the initial ratios are uncorrelated (or negatively correlated) with the changes that now occur in the values of the Slutsky substitution terms relating to X_i, the first-order conditions define an optimum with respect to the X_i-ratio. The optimum is local if (but not only if) this requirement is met in the vicinity of the value of the X_i-ratio prescribed by the first-order conditions; it is global if (but not only if) it is met for all changes in the X_i-ratio.[27]

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[26] H. A. John Green, "The Social Optimum in the Presence of Monopoly and Taxation," Review of Economic Studies, Vol. 29 (October, 1961), pp. 66-78.

[27] Ibid., p. 77.

Like those of Lipsey and Lancaster, these results of Green's are inconclusive in the sense that determinate solutions for the location of an optimum, given the preclusion of a Paretian optimum, are not very easy to discover.

A further problem with regard to the practical applicability of the theory of second-best is that concerned with the question of the existence of a true second-best solution. Aside from the fact that each of the many possible combinations of constraints, in addition to the usual constraint assumed operative in the determination of the Paretian optimum, yield a second-best solution,[28] there is the problem that each one of these possible combinations of constraints may give rise to more than one second-best optimum. Because of the complexity of the set of first-order conditions defining a second best optimum, a true second-best solution may not be capable of achievement. For these first-order conditions, as expressed by equation (17), involve knowledge of the second-order derivatives of the utility and transformation functions. The behavior of these second order derivatives is unknown.

When consideration is given to the second-order conditions for the second-best optimum the difficulties involved in finding a true second-best solution are "quite insurmountable."[29] These second-order or stability conditions refer to the convexity of the community's preference function and

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[28]Lipsey and Lancaster, "The General Theory of Second Best," p. 13.

[29]*Ibid.*, p. 28.

the concavity of the transformation function; and, in combination, ensure the attainment of a maximum, rather than a minimum, position. They may be set out as follows:

$$d^2L = (F_{ij} - u'\emptyset_{ij} - v\,H_{ij})\, d_{xi}d_{xj} +$$

$$(20) \qquad (F_i - u'\emptyset_i - vH_i)\, d^2x_i$$

$$= (F_{ij} - u'\emptyset_{ij} - v\,H_{ij})\, dx_i dx_j < 0 \;;$$[30]

that is, the second-order differential of the maximization problem expressed by equation (11) must be less than zero. Because this second-order differential involves third-order derivatives, the second-order conditions are of no help in the determination of a true second-best solution. For, "absolutely nothing is known" about these third order derivatives of F and Ø in the general case and "their properties cannot be derived from the second-order condition that the Paretian optimum represents a true maximum for F."[31]

The discussion of the last few paragraphs with regard to the possibility of defining a true determinate second-

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[30] McManus, "Comments on the General Theory of Second Best," p. 220. McManus also notes the fact that, strictly speaking, the attainment of a maximum is ensured subject not only to necessary satisfaction of equation (20) but also to the condition that changes in the quantities of the various goods produced, dx_i, be restricted by the requirement that the first differentials of equations (2) and (9) equal zero, ibid., pp. 220-1.

[31] Lipsey and Lancaster, "The General Theory of Second Best," p. 28.

best solution given the insurmountable difficulties involved in specifying the properties of the first- and second-order conditions raises certain undeniable doubts about the practical applicability of the general theory of second best.[32] In the first instance, the results of the Lipsey-Lancaster analysis, which culminated in the general theory of second-best, might be conceived of as dealing a death-blow to the piece-meal application of welfare economics. However, as yet no criteria for judging policy have been developed directly from the results of the general theory of second-best that would serve as a replacement for the criteria derivable from the conditions defining a Paretian optimum. For, indeed, the main conclusions of the general theory of second-best take the form of negative corollaries which effectively serve to classify this theory as an "impossibility" theorem.

In the face of the impossibility-type analysis of the general theory of second-best, the question, then, to be asked is whether or not the Paretian conditions may yet serve as a guide to practical policy. The general theory of second-best offers no practical guide to the way in which the second-best optimum can be achieved. It is

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[32] Green, loc. cit., pp. 69-77, discusses with the help of matrix algebra the behavior and applicability of the first-order and second-order conditions derived from his attempt to find a social optimum in the presence of monopoly and taxation; but, like Lipsey and Lancaster, is unable to specify the properties of these condition.

possible, therefore, that the Paretian conditions may serve as better criteria than no criteria at all. Accordingly, it is to the question of the application of the Paretian conditions as a guide to policy and, specifically, to the application of a "third-best" type analysis that attention is directed in the final chapter.

CHAPTER V

PRAGMATIC THIRD BEST ANALYSIS

In Chapter I the attainment of a Paretian optimum was shown to require the simultaneous universal satisfaction of a set of first order marginal conditions. It was then shown in Chapter II that the simultaneous satisfaction of these marginal conditions effectively required perfect knowledge of the economic system and perfect power to make use of this knowledge, that is perfect power to implement the equality of the several marginal conditions. In the real world, however, the existence of certain irremovable constraints which cause divergences between certain marginal values and costs constitutes the absence of perfect power. The results of several attempts to derive an optimal solution subject to the assumed operation of constraints in addition to the usually operative transformation function were surveyed in Chapter III and were shown to be similar. Consideration of these results had led to the setting out by Lipsey and Lancaster of the General Theory of Second Best.

The General Theory of Second Best and its main conclusions were briefly set out in Chapter IV. It was noted, however, that the set of marginal first-order conditions defining the second-best optimum were much more complex than

those defining a Paretian optimum. Indeed, the amount of information required to locate a determinate second-best solution by means of its necessary conditions is in general outside the realm of practicability. Effectively, the attainment of a second-best optimum requires perfect knowledge. But in the real world there is no entity which possesses perfect knowledge. Accordingly, the important practical conclusions of the general theory of second-best take the form of negative corollaries whereby the policy maker is left in a quandary without criteria by which to judge policy proposals.

While admitting "that in general one can say nothing in the absence of universal optimization; further, that the vast and intricate knowledge required in order to derive quantitatively exact second-best solutions will be denied to us in the foreseeable future,"[1] E. J. Mishan, nevertheless attempts to set out certain conditions which would prove amenable to the piece-meal application of welfare economics. That is, rather than embrace the conclusions of the general theory of second best as "an anodyne against further thought,"[2] he attempts to set out the circumstances which would permit the use of the Paretian conditions as sufficient conditions for an increase in welfare even though

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[1]E. J. Mishan, "Second Thoughts on the Second Best," Oxford Economic Papers, Vol. 14 (October, 1962), p. 205.

[2]Ibid., p. 216.

all the Paretian conditions are not capable of being satisfied simultaneously and, thereby, that the attainment of a Paretian optimum is precluded. Accordingly, Mishan sets out certain general a priori statements as follows:

> (1) it would seem very reasonable to believe that (i) the smaller are the constrained sectors relative to the remaining ones, and (ii) the larger are the initial discrepancies in the price-marginal cost ratios of the free sector as compared with the constrained sectors, the surer we are to improve matters by optimizing in the free sectors alone than by standing by and sadly sucking our thumbs under the sign of second best.
>
> (2) the larger are the differences in price-marginal cost ratios as between industries in the free sectors relative to these differences in the constrained sectors, the more certain we are to improve matters--to come closer to an ideal allocation--by adopting as a rule for all the free sectors any one of the price-marginal cost ratios in the constrained sectors.
>
> (3) the price-marginal cost ratio to be adopted by the free sectors would not be far removed from a correct second-best solution if it were calculated as an average of the ratio of the constrained sectors, each such ratio being weighted by the value added of that sector.[3]

These rough rules-of-thumb with which the profitable application of a partial-Paretian optimum rule can be justified may be thought of as representing a "third-best" analysis. That is, given that a Paretian optimum is unattainable because of imperfect power to simultaneously satisfy all the Paretian conditions and given, furthermore, that a second-best solution is unattainable because of the absence of perfect knowledge by which to determine the magnitude and direction

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[3] _Ibid._, pp. 213-5.

of the necessary and sufficient conditions of such a second best solution, a special type of third-best solution may be thought to be capable of attainment by the use of the Paretian-optimal conditions as sufficient conditions given the existence of certain specified circumstances.

In the field of international trade and particularly with regards to the formation of customs unions and the discriminatory reduction of tariffs, there has been just such a development of rough rules-of-thumb by which the application of Paretian welfare economics may be guided. In Chapter IV the analyses of Meade, Osga, Viner and Lipsey and Lancaster were surveyed and their second-best conclusions noted. From these second best conclusions attempts have been made by Meade[4] and Viner,[5] in particular, to set out the circumstances in which this specific type of third-best analysis could be applied, that is, the circumstances in which the application of the Paretian optimal rule would result in a closer approximation of an ideal allocation of resources and a move towards a free trade position defined as that in which the welfare of the world community would be maximized.

Now, according to Viner, favorable conclusions with regard to the benefits derivable from the partial freeing of

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[4]J. E. Meade, The Theory of Customs Union (Amsterdam: North Holland Publishing Co., 1955).

[5]Jacob Viner, The Customs Union Issue (New York: Carnegie Endowment for International Peace, 1950).

trade results from the inadequate consideration of at least three problems.[6] There is first a failure on the part of some writers to consider adequately the effects arising from the extension, by the formation of a custom's union, of the area within which protection is given to certain domestic producers by the tariff level originally levied by one of the member countries. That is, subject to the condition that country A initially levies a high rate of tariff on the imports of commodity X from country C and to the condition that country B, who also imports X from country C, levies a lower rate of tariff on imports of X; then, the formation of a customs union between countries A and B will probably result in the extension of the pre-existing high rate of tariff levied by country A on her imports of X to the imports of X by country B. A second problem which leads to confusion is that which results from the lack of precision given to the differentiation of the effects of the formation of a customs union on the location of industry from the effects on the location of the burden of the import duties. And, thirdly, there is generally the tendency to consider the effects on only the member countries and, if these effects are favorable, to conclude that world welfare is increased by the partial freeing of world trade as would result from the formation of a customs union.

After giving, what he believes to be sufficient,

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[6] Ibid., p. 53, n. 12.

consideration to these above-mentioned problems, of which inadequate consideration has led other writers to expound the virtues of free trade, Viner, himself, arrives at certain tentative conclusions with regard to the favorableness of the formation of a customs union and, thereby, of the removal of certain constraints which prevent the satisfaction of certain of the marginal conditions. The welfare effects of the formation of a customs union on the welfare of the member countries and on the rest of the world may be found by examination of several factors, among which are the extent of coverage of the customs union, the average rate of tariffs or imports by the proposed member countries, the extent to which the member countries produce complementary products, the size of firms and industries within the various member countries, the extent to which the specialization of production be enhanced by the formation of the customs union as between the member countries and the remainder of the world trading community, and the ability of previously protected industries within the various member countries to take advantage of the increased size of their protected market. Examination of these factors yields the conclusion that in general the formation of a customs union will have a greater chance of increasing the welfare of the member countries and of the whole world:

(1) the larger the economic area of the customs union and therefore the greater the potential scope for internal division of labor;

(2) the lower the "average" tariff level on imports from outside the customs union area as compared to what the level would be in the absence of customs union;

(3) the greater the correspondence in kind of products of the range of high cost industries as between the different parts of the customs union which were protected by tariffs in both of the member countries before customs union was established, i.e., the less the degree of complementarity--or the greater degree of rivalry--of the member countries with respect to protected industries, prior to customs union;

(4) the greater the differences in unit-costs for protected industries of the same kind as between the different parts of the customs union, and therefore the greater the economies to be derived from free trade with respect to these industries within the customs union area;

(5) the higher the tariff levels in potential export markets outside the customs union area with respect to commodities in whose production the member countries of the customs union would have a comparative advantage under free trade, and therefore the less injury resulting from reducing the degree of specialization in production as between the customs union area and the outside world;

(6) the greater the range of protected industries for which an enlargement of the market would result in unit-costs lower than those at which the commodities concerned could be imported from outside the customs union area;

(7) the smaller the range of protected industries for which an enlargement of the market would not result in unit-costs lower than those at which the commodities concerned could be imported from outside the customs union area but which would nevertheless expand under the customs union.[7]

These conclusions of Viner, then, may be thought of as a third-best type analysis by which the policy maker may be guided in

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[7] Ibid., pp. 51-2.

his attempt to reach an optimal free-trade position of maximum welfare through the piece-meal application of welfare economics and, specifically, by the formation of a customs union or the reduction of tariffs where possible.

In a similar manner, J. E. Meade has set out certain rough rules-of-thumb by which a policy maker might be guided in his attempt to achieve a global position of maximum welfare. Meade has concluded[8] that the formation of a customs union will more than likely raise welfare: if the possibilities of primary trade expansion are widespread; if there is a great potential complementariness of productive facilities even though at the outset the two countries are competitive in their production; if the possibilities of expanding their mutual trade are great, as would be the case given initial high rates of tariff on the goods they import from one another; if the member countries are to one another their own principal suppliers of imported commodities; if the proportion of the world's population and trade incorporated by the customs union is large; if the level of tariffs in the rest of the world is low and the number of customs unions into which the remaining countries of the world are divided is large; if the barriers to world trade mainly take the form of fixed quantitative controls and quotas rather than tariffs; and, finally, if the possibilities of developing large scale economies of production within the member countries are widespread. These conditions as set out by Meade, then,

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[8]Meade, loc. cit., pp. 107-15.

are seen to be most similar to those set out by Viner; and, once again, may be thought of as being part of a third-best type analysis. For even though the attainment of both a Paretian optimum and a second-best optimum is precluded, the existence of these general conditions is such as to make it possible for the partial application of the Paretian optimum conditions, specifically, for the partial removal of tariffs to lead towards a position of maximum world welfare.

In another field of economics, that of highway construction, pricing and utilization, a third-best type of analysis has also been developed. After analyzing the optimum scale of new highway construction, the optimum level of highway use and the optimum pricing system for highways according to the criteria of the Paretian optimum marginal conditions. D. M. Winch in an appendix to his forthcoming book, The Economics of Highway Planning,[9] shows that, given the absence of perfect power and perfect knowledge and, thus, the impossibility of obtaining either a Paretian optimum or a second-best optimum, the partial application of Paretian welfare economics does not invalidate his conclusions about the optimum levels of highway construction, use and pricing. In essence, Winch concludes that the circumstances, involved in the problems of the allocation of resources to the building

[9] D. M. Winch, The Economics of Highway Planning (Toronto: Toronto University Press, to be published), pp. 399-407.

of highways, of the optimum use of these highways once built, and of the financing of these highways, are such as to allow one to invoke the use of partial-Paretian optimum analysis thereby recognizing, but treating as insignificant or in total neutral, the effects of such analysis on the remaining sectors of the economy.

Mention should be made here of the fact that Winch's analysis itself contains two methods of approach to the problem of optima in a multi-constrained setting. Rather than concerning oneself with the original set of marginal equalities derived from a singly constrained situation, Winch so broadens his definitions of these marginal equalities as to include the effects of additional constraints. That is, he works with the all-inclusive terms of marginal social cost and marginal social benefit. Accordingly, his concluding discussion of the implications of the General Theory of Second Best on his main analysis of the highway sector is perhaps unnecessary; for he has in fact already discussed in part the impact of the constrained economy on the problems of highway planning by his explicit use of the broadly defined terms marginal social cost and marginal social benefit.

To the problem of determining a doubly constrained optimum there are thus two general methods of approach, the inadequate differentiation of which can lead to confusion. First, one can concentrate on the new marginal equalities derived from maximization subject to additional constraints.

These new marginal equalities may be interpreted as representing the broadly defined marginal social cost and marginal social benefit. These broadly defined equalities thus take on a different quantitative form when compared to the original equalities. Alternately, one can concentrate on the original set of marginal equalities derived from maximization subject to the usually postulated constraint. In this case the results of the operation of additional constraints are examined with reference to the original marginal equalities and to the magnitude and direction of the necessary divergences in these original marginal equalities in order that they be used as guides for movements towards a maximum solution. It is with this last approach to the problem of additionally constrained optima that the General Theory of Second Best is concerned and with which this thesis has been primarily concerned.

The above examination of the attempts by Meade and Viner in the field of international trade and by Winch in the field of highway construction to overcome the impossibility conclusions of the general theory of second-best are instructive in the setting out of a method of approach to the problem of welfare maximization given the preclusion of the attainment of both a Paretian optimum and a second-best optimum. However, it might be noted that the above-mentioned analysis differ somewhat in their approach. For in the problem of highway planning, construction, use and pricing, the approach took the form of applying the Paretian conditions for an optimum

in this particular sector. The results obtained from this partial application of Paretian welfare economics were then examined to see whether or not the repercussions throughout the remaining sectors of the economy were great enough to counterbalance the otherwise obtainable gains within the highway sector or whether or not imperfections in the remaining parts of the economy balanced each other out. The conclusions of Winch's analysis, noted above, justify the partial application of the Paretian conditions in the highway sector.

In general, the Paretian conditions for an optimum could be applied to any particular maximization problem, including those problems concerned with the optimum levels of public expenditure and investment and those concerned with broader welfare maximization at the national level or at the international level. Following Winch's approach, the Paretian conditions would be applied to any one of these various types of maximization problems where it was possible to apply them. The anticipated results of any such hypothetical partial application of the Paretian conditions would then be examined and the repercussions throughout the economy checked. In particular, primary concern would be directed towards the effects on the allocation of resources.

The partial application of the Paretian conditions for an optimum would, then, be justified if the repercussions throughout the remaining parts of the economy were negligible

and, thus, could be explicitly neglected. The increased welfare resulting from the maximization of the particular welfare level would, then, be considered as the determining factor guiding the policy maker in his attempt to maximize welfare. And, secondly, if the repercussions are significant but of a favorable type, then, the partial application of the Paretian conditions for an optimum is, likewise, justified. If these repercussions are significant but of an unfavorable type or if they are unknown, then it may be concluded that the Paretian conditions are unsuitable as sufficient conditions for a movement towards a maximum position and, thereby, are unsuitable as adequate criteria by which policy proposals could be judged.

In the problem of achieving the obtainable optimum allocation of resources in any one economy, this above-discussed method of approach would take the form of postulating the equating of price to marginal cost in any industries that were nationalized. The expected repercussions throughout the remaining sectors of the economy, in particular, the effect on the allocation of resources would then be examined in the afore-mentioned manner.

This essentially pragamatic approach to the maximization of welfare at the national level may also be taken at the international level with respect to the optimum tariff system and, thereby, the optimum allocation of production as between the various countries of the world. The expected results of any customs union or discriminatory reduction of tariffs

would have to be examined. The results of such an examination would then either justify or not justify the application of piece-meal Paretian welfare economics, that is, the formation of a customs union or the discriminatory reduction of tariffs.

In any of these welfare maximization problems, it may be found that the application of the Paretian conditions for an optimum would lead away from the obtainable optimum. That is, at the national level and with regard to the optimum pricing policy for a nationalized industry, it may be found that the Paretian conditions of price-marginal cost equality would only cause the allocation of resources to be further distorted. Similarly, in the problem of determining the optimum system of tariffs, it may be found the repeal of any one tariff or the formation of a particular customs unions leads away, rather than towards, the obtainable optimum allocation of production among the countries of the world.

A slightly different method of approach is evidenced in the third-best type analyses of Meade and Viner concerning the welfare effects of the formation of a customs union. In these analyses emphasis is placed on the circumstances which must necessarily prevail before the partial-application of the Paretian conditions for an optimum would be justified as sufficient conditions for a movement towards a position of maximum welfare. In their approach, Meade and Viner follow closely the method of approach outlined by Mishan,

who felt that "we may still be able to indicate certain easily conceived conditions that permit us to say something useful; in particular we may be able to discover circumstances which enable us to derive guidance from the familiar optimum rules even though these rules are not universally met."[10] In any particular field of economics and concerning any particular problem of welfare maximization it should be possible to set out general conditions under which the partial application of the Paretian optimum conditions could be justified guides for an increase in welfare. Once this set of general conditions has been developed, the then remaining question to be answered by the policy maker is that concerned with whether or not this set of conditions prevails. For as already mentioned there will be certain welfare maximization problems which just do not lend themselves to the piece-meal application of Paretian-welfare economics.

A general method of approach to the problem of maximization subject to the impossibility of attaining either a Paretian optimum or a second-best optimum is thus suggested. In any such welfare maximization problem of any particular sector a partial-equilibrium-type analysis may be pursued by applying the Paretian conditions for an optimum where it is possible to apply these. The results of this application may then be examined with respect to the repercussions on the remaining sectors of the economy, and, specifically,

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[10]E. J. Mishan, p. 205.

these repercussions may be shown to be negligible, significant but of favorable consequences, significant but of unfavorable consequences or unknown. If these repercussions are of either of the first two types, that is, either negligible or significant but favorable, then, it may be concluded that the Paretian marginal conditions will serve as rough but sufficient guides for an increase in welfare, and the solution thereby obtained may be referred to as a "third-best" solution.

From the frequent partial application of the Paretian conditions for an optimum to any one maximization problem, there will result the gradual development of a set of general conditions which may be used in subsequent policy decisions as rough rules-of-thumb whereby the policy maker can decide quickly and without recourse to detailed analysis whether or not the Paretian conditions can be used advantageously as sufficient criteria for the guiding of policy. The two methods of approach are thus seen to blend together into a general method of approach.

Of primary concern to the national or international policy maker, however, is the establishment of a general set of conditions which, if present, would require a third-best solution. Accordingly, the problem resolves into one of finding those circumstances which prove amenable to a third-best type analysis, and of setting out the conditions which describe such circumstances and which would guide the

policy maker. In this line, Meade and Viner have set out the conditions which if present in the world community would permit the partial application of the Paretian conditions, namely, the partial freeing of trade to lead to an optimal solution.

In a similar manner, at the national level the problem of the optimum pricing policy to be pursued by a nationalized industry could be examined and from such an examination there could be developed a set of general conditions which, if present, would call for a third-best solution. Now in Chapter IV this problem, as set out in a three-good three-industry model and as analyzed by Lipsey and Lancaster to illustrate the conclusions of their general theory of second-best, was shown to yield a solution which required, according to certain strict assumptions, that the price of the nationalized industry must be set higher than its marginal cost but not quite as high as the monopolist set the price of his product in relation to its marginal cost. For our purposes here, however, it is expedient to remove the restrictions of there being only three industries. Accordingly, it is assumed there are several industries making up the economy, some of which are monopolies in which there is a divergence between the price and marginal cost of their products and some of which are perfectly competitive industries in which price equals marginal cost.

It is further postulated that there are two industries

which produce fuel for heating purposes. One of these industries is assumed to be a monopoly producing, say, gas for which the price is above the marginal cost; the other industry was a monopoly producing, say, electrical power but has been nationalized. Now a Paretian optimum is unattainable and, according to the general theory of second-best, the Lerner Rule for equating price to marginal cost is strictly speaking unsuitable. That is, a second-best solution will be possible only if price deviates from marginal cost to some specified extent. However, the information required to determine a true second best solution is not available and, thereby, like the Paretian optimum, the second-best optimum is assumed incapable of satisfaction. The question now posed is: "What are the general conditions, whose existence would specify the optimum pricing policy of the nationalized industry as one of a third-best type calling for the equality of the price of electrical power and its marginal cost?".

An examination of the posed problem yields the following conclusions: namely, that the third-best type pricing policy is more likely to increase welfare and lead to a more ideal allocation of resources:

(1) the lower the rate of divergence between prices and marginal costs throughout the industries other than these two fuel-producing industries; for the consumption of fuel all of which had a

high price in relation to its costs will now be increased at the expense of the products of other industries whose price was lower in relation to marginal cost. That is, before nationalization, other products may have been substituted for fuel consumption;

(2) the lower the rate of divergence between the price and marginal cost of gas; for, as has been shown before, if there is a high rate of divergence in the gas-producing industry, the equating of price to marginal cost in the electric-power producing industry might well have the effect of diverting business away from a socially inexpensive gas-producing industry to a socially expensive electrical-power producing industry;

(3) the more highly elastic the demand for electricity, and the lower the cross elasticity of demand for gas with respect to a change in the price of electricity; and accordingly, if these elasticities are high enough and low enough respectively, the equating of price to marginal cost in the electrical power producing industry will result in increased consumption of electrical power by attracting new consumers and not by causing a decrease in consumption of gas (a change in the pricing policy of the

gas producers is not considered here);

(4) the greater is the rate of divergence between the price of electrical power itself and its marginal cost;

(5) the larger is the electrical-power producing industry compared to the rest of the economy.

If these conditions prevail in their entirety or indeed if they exist only in part, the Paretian rule for the attainment of an optimum should be applied in the analysis of this particular problem. The repercussions may then be examined to check further as to whether or not the piecemeal application of Paretian welfare economics would in fact be justified, that is, whether or not such an application of the Paretian conditions would in the particular maximizing problem analyzed lead towards the obtainable optimum.[11]

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[11] It may be possible at this point to further distinguish between a third-best solution and a fourth best solution. In this problem of the optimum pricing system to be followed by the nationalized electrical-power-producing industry, the existence of a certain set of general conditions is thought to justify the partial application of the Paretian optimum rule, namely, the equation of price to marginal cost. This partial application, justified by the existence of a general set of conditions may be thought to represent a fourth best analysis. But in fact, the existence of this general set of conditions is neither a necessary nor a sufficient condition. Accordingly, a true third best solution of the type obtained from the partial application of the Paretian rule is only found when the expected repercussions of the postulated partial application of the Paretian optimum rule are found to be favorable.

In a similar manner there could be developed, with reference to any one maximization problem in the face of additional constraints, a set of general conditions which would guide the policy maker to a third best solution of a type other than that obtained from the partial application of the Paretian conditions. That is, there could be developed a set of general conditions whose existence would guide the policy maker, in his attempt to find the optimum pricing system for a nationalized industry in a mixed economy, to set the price-marginal cost ratio of the nationalized good equal to, say, the average price-marginal cost ratio existing throughout the imperfectly competitive sectors of the economy. There will be in practice a large number of third-best optimum rules which correspond to the existence of different sets of conditions, which conditions if prevalent throughout the economy would justify the use of the particular third-best optimum rule in the attempt to achieve a third best solution. It should thus be emphasized that the third-best type analysis discussed in this chapter is only one of many possible such analyses.

The essence of a third-best type analysis is thus seen to be the pragmatic way in which the obtainable optimum position is found. Given the preclusion of a Paretian optimum because of imperfect power and the preclusion of a second best optimum because of imperfect

knowledge with which to specify the conditions defining a second best optimum, there is then an obtainable optimum which may or may not be approached by means of the Paretian conditions as policy guides. A certain number of obtainable optima will, however, be found as a result of the piecemeal application of the Paretian optimum conditions. This particular type of obtainable optimum has been called a third-best optimum.

In sum, there will be certain welfare maximization problems which, when examined, are shown to be of the type to which the partial application of Paretian welfare economics will serve as the best method of obtaining an optimum position or a movement towards an optimum position. That is, there will be certain maximization problems, characterized by the existence of certain conditions, to which the application of the Paretian optimum rule where possible will serve as the best policy for a movement towards the obtainable optimum. And finally, there will be certain maximization problems which definitely do not lend themselves to the partial application of the Paretian conditions. That is, the use of the Paretian optimum conditions as policy guides would result in a movement away from, rather than towards, the obtainable optimum position. Accordingly, some deviation from the Paretina optimum condition of, say, price equal to marginal cost will be necessary in the problem of defining the optimum

pricing policy in a mixed economy. As in the Lipsey-Lancaster model of the nationalized industry, the obtainable optimum will be more nearly approximated by allowing the nationalized industry to produce at a level at which price is somewhat above marginal cost.

The first category of maximization problems is probably the most important. Accordingly, at the international level, as Meade and Viner have done, a set of general conditions may be spelled out which, if present, in whole or in part, would suggest that the application of the Paretian optimum conditions[12] as sufficient conditions will not prove disasterous but rather, in the absence of any other criteria, will guide the policy maker in his attempt to discover the obtainable optimum position. And similarly, at the national level, as Mishan has already done in part, a set of conditions may be laid out whereby the optimum pricing policy of a nationalized industry, which would lead towards the obtainable optimum allocation of resources in the economy is said to be one requiring the equating of marginal cost to price.

The Paretian conditions may thus yet serve in certain circumstances and with regard to certain constrained maximization problems as sufficient criteria for an increase in welfare and a movement towards the optimum position.

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[12] In this problem, the condition of a Paretian optimum with which the policy maker is concerned is that of the absence of tariffs.

The negative conclusions of the General Theory of Second Best are, accordingly, somewhat mollified. The main contribution of the General Theory of Second Best is now seen to be that of providing an amber light with regard to the hasty and haphazard use of the Paretian conditions as criteria with which policy can be judged and also that of emphasizing the necessity of approaching public policy at the industry, national or international level in a pragmatic manner.

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